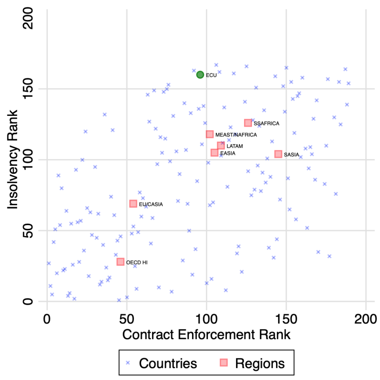
Supplemental Material (Not for Publication)

SM1 Additional Descriptive Statistics

Supplemental Material Figure SM1 shows the position of Ecuador in terms of contract enforcement and insolvency in the World Bank Doing Business report. Lower numbers represent better institutions to enforce contracts or solve insolvency cases.

Figure SM1: **Ranks Insolvency and Enforcement**



Notes: This figure presents the location of Ecuador in the World Bank Doing Business ranks in the categories of Insolvency (Y-Axis) and Enforcement (X-Axis). Most efficient country in terms of enforcement ranks 1st.

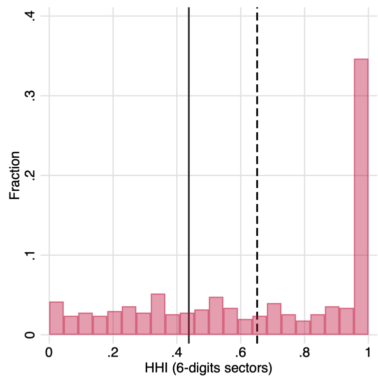
Supplemental Material Figure SM2 shows the distribution of Herfindahl-Hirschman Indices (HHI) for manufacturing 6-digit sectors in 2017. HHI_s for sector s is estimated using the following formula:

$$HHI_s = \sum_{j \in J_s} m_j^2,$$

where m_j is the market share of firm j, J_s is the set of active firms in sector s. The market share of firm j is obtained by dividing total revenue of firm j by the sum total revenue of all firms in sector s.

Supplemental Material Figure SM3 presents the cumulative distribution of quantities, both in relative terms through standardized quantities and in absolute values through residualized log quantities, for different age of relationships.

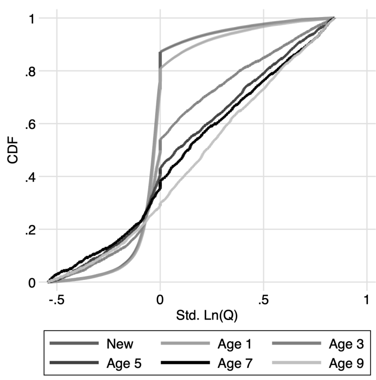
Figure SM2: **Distribution of Herfindahl-Hirschman Indices for Manufacturing in 2017**



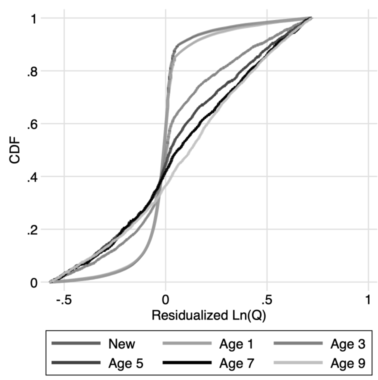
Notes: This figure presents a histogram of estimated Herfindahl-Hirschman Indices (HHI) for 6-digit manufacturing sectors in 2017.

Figure SM3: **Cumulative Distribution Function of Quantities by Relationship Age**

(a) Product-level Standardized Ln(Q)



(b) Seller-level Residualized Ln(Q)



Notes: These figures plot the cumulative distribution functions for standardized log quantities (left) and residualized log quantities (right) by different ages of relationship.

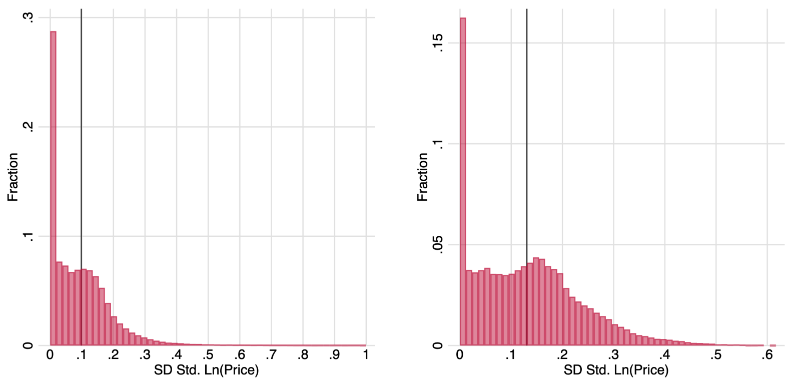
SM1.1 Price and Quantity Dispersion

Supplemental Material Figures SM4 and SM5 show the dispersion of standardized log prices and quantities, respectively.

Supplemental Material Figure SM4 shows that the average product has an average standard deviation of prices close to 0.10. This implies that in a given, the same product could have prices that are 10% higher or lower than the average price more than 30% of the time. Similarly, Supplemental Material Figure SM5 shows that the average standard deviation of quantities for a given product in a month is close to 0.4.

Figure SM4: **Product-level Price Dispersion within Month and Year**

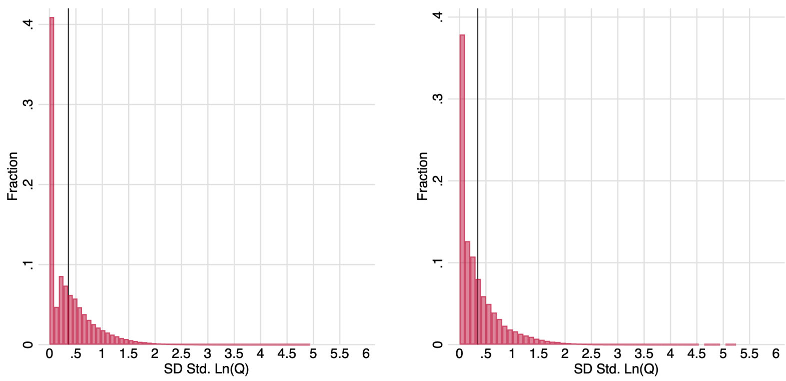
(a) Month (b) Year



Notes: These figures plot histograms of the standard deviation of standardized log prices by month and year, for products that have at least 5 distinct buyers in time window.

Figure SM5: **Product-level Quantity Dispersion within Month and Year**

(a) Month (b) Year



Notes: These figures plot histograms of the standard deviation of standardized log quantity by month and year, for products that have at least 5 distinct buyers in time window.

SM2 Solution of Gamma Function for Stationary Equilibrium

The seller's first order condition defines the following differential equation in the stationary equilibrium

$$\theta u'(q(\theta)) - c = \frac{\Gamma(\theta) - F(\theta) + (1-\delta)\theta\gamma(\theta)}{f(\theta)} u'(q(\theta)). \tag{47}$$

The solution $\Gamma(\theta)$ to the equation above is given by:

$$\Gamma(\theta) = \frac{\int_{\underline{\theta}}^{\theta} x^{\delta/(1-\delta)}[xf(x) - c(u'(q(x))^{-1}f(x) + F(x)]dx + K}{\theta^{1/(1-\delta)}(1-\delta)}, \tag{48}$$

which by integration by parts reduces to:

$$\Gamma(\theta) = \frac{F(\theta)}{1-\delta} - \frac{\delta\int^{\theta} x^{\delta/(1-\delta)}F(x)dx}{(1-\delta)\theta^{1/(1-\delta)}} - \frac{cE[x^{\delta/(1-\delta)}u'(q(x))^{-1}|x \leq \theta]}{(1-\delta)\theta^{1/(1-\delta)}} + \frac{K}{(1-\delta)\theta^{1/(1-\delta)}} \tag{49}$$

The constant is obtained by using the boundary condition $\Gamma(\bar{\theta}) = 1$. Therefore,

$$K = cE[x^{\delta/(1-\delta)}u'(q(x))^{-1})] - \delta\bar{\theta}^{1/(1-\delta)} + \delta\int x^{\delta/(1-\delta)}F(x)dx. \tag{50}$$

SM3 Proofs - Model Dynamics

Proof of Proposition 1. Recall the quantity function $q_\tau(\theta)$ and its inverse function $\theta_\tau(q)$. Further differentiating the derivative of the incentive-compatible tariff schedule $T'_\tau(q_\tau(\theta)) = \theta v'(q_\tau(\theta))$ gives:

$$T''_\tau(q) = \theta'_\tau(q)v'(q) + \theta_\tau(q)v''(q) = \theta(q)v'(q)\left[\frac{\theta'_\tau(q)}{\theta_\tau(q)} + \frac{v''(q)}{v'(q)}\right] \tag{51}$$

$$= T'(q)\left[\frac{1}{\theta_\tau(q)q'_\tau(\theta)} - A(q)\right], \tag{52}$$

for $A(q) = -v''(q)/v'(q)$ and $\theta'_\tau(q) = 1/q'_\tau(\theta)$.

By implicit differentiation on the seller's first-order condition Number we obtain an expression for $q'_\tau(\theta)$:

$$q'_\tau(\theta) = -\frac{\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]v'(q_\tau(\theta))}{\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]v''(q_\tau(\theta))}$$

$$= \frac{1}{A(q_\tau(\theta))}\frac{\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}{\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}$$

The denominator of the equation above is positive as $v'(q_\tau(\theta)) > 0$ and $c > 0$. As by assumption, strict monotonicity holds $q'_\tau(\theta) > 0$, then the numerator is also positive. Substituting in 51 and using the fact that $T'_\tau(q) > 0$ and $A(q_\tau) > 0$, quantity discounts $T''_\tau(q) \leq 0$ hold if and only if

$$\frac{\left[\theta - \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}{\theta\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]} \leq 1 \tag{53}$$

Define the $\Lambda_\tau(\theta) \equiv \Gamma_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)$ and $\lambda_\tau(\theta) \equiv d\Lambda_\tau(\theta)/d\theta$. Inequality 53 holds if

$$\theta - \frac{\Lambda_\tau(\theta) - F_\tau(\theta)}{f_\tau(\theta)} \leq \theta - \theta\frac{(\lambda_\tau(\theta) - f_\tau(\theta))f_\tau(\theta) - (\Lambda_\tau(\theta) - F_\tau(\theta))f_\tau'(\theta)}{f_\tau(\theta)^2}.$$

Rearranging, one obtains

$$[\Lambda_\tau(\theta) - F_\tau(\theta)][f_\tau(\theta) + f_\tau'(\theta)\theta] \geq \theta f(\theta)[\lambda_\tau(\theta) - f_\tau(\theta)]. \tag{54}$$

As noted above, $\theta f_\tau(\theta) \geq \Lambda_\tau(\theta) - F_\tau(\theta)$. Note that log-concavity of the density $F_\tau(\theta)$ is sufficient to satisfy the assumption of monotone hazard condition. For log-concave densities, the following inequality holds $f_\tau(\theta) \geq f_\tau'(\theta)\theta$. Therefore, if $\Lambda_\tau(\theta) > F_\tau(\theta)$, then a sufficient condition for quantity discounts is $\lambda_\tau(\theta) < f_\tau(\theta)$.

Instead if $\Lambda_\tau(\theta) < F_\tau(\theta)$, one can write 53 as

$$(\theta - 1)f_\tau(\theta) + f_\tau(\theta) \geq [F_\tau(\theta) - \Lambda_\tau(\theta)]\left(1 + \frac{f_\tau'(\theta)\theta}{f_\tau(\theta)}\right) + \lambda_\tau(\theta). \tag{55}$$

If $f_\tau'(\theta) < 0$, then a sufficient condition is $(\theta - 1)f_\tau(\theta) \geq F_\tau(\theta)$. If $f_\tau'(\theta) > 0$, then a sufficient condition is that $(\theta - 1)f(\theta) \geq F_\tau(\theta)(1 + \theta f_\tau'(\theta)/f_\tau(\theta))$, which can be expressed as:

$$\frac{d}{d\theta}\left(\frac{F_\tau(\theta)}{f_\tau(\theta)}\right) = \frac{f_\tau(\theta)^2 - F_\tau(\theta)f_\tau'(\theta)}{f_\tau(\theta)^2} \geq \frac{F_\tau(\theta)}{(\theta - 1)f_\tau(\theta)}. \tag{56}$$

□

Proof of Proposition 2. Notice that by the seller's first-order condition and $v'(\cdot) > 0$, $q_\tau(\theta) \leq q_{\tau+1}(\theta)$ holds if and only if

$$\begin{aligned} V_\tau(\theta) &\equiv \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)} \\ &\geq \frac{f_\tau(\theta)}{f_{\tau+1}(\theta)}\frac{\Gamma_\tau(\theta) - F_{\tau+1}(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_{\tau+1}(\theta)}{f_\tau(\theta)} + \frac{\Gamma_{\tau+1}(\theta) - 1}{f_{\tau+1}(\theta)}, \end{aligned}$$

which can be written as

$$V_\tau(\theta) \geq \frac{f_\tau(\theta)}{f_{\tau+1}(\theta)}V_\tau(\theta) + \frac{\Gamma_{\tau+1}(\theta) - 1}{f_{\tau+1}(\theta)} + \frac{\theta[\gamma_{\tau+1}(\theta) - \gamma_\tau(\theta)]}{f_{\tau+1}(\theta)} - \frac{F_{\tau+1}(\theta) - F_\tau(\theta)}{f_{\tau+1}(\theta)}.$$

With no selection pattern, i.e. $f_\tau(\theta) = f_{\tau+1}(\theta)$, the condition reduces to

$$\frac{1 - \Gamma_{\tau+1}(\theta)}{f_\tau(\theta)} \geq \frac{\theta[\gamma_{\tau+1}(\theta) - \gamma_\tau(\theta)]}{f_\tau(\theta)}.$$

As $\gamma_\tau(\theta) > 0$ by assumption and the left-hand side is (weakly) positive due to $\Gamma_{\tau+1}(\theta) \leq 1$, a sufficient condition is that $\gamma_{\tau+1}(\theta) < \gamma_\tau(\theta)$. To obtain necessity, consider the Lagrangian keeping future return U^+ constant. The seller chooses $q(\theta)$ maximizing the following program:

$$L(\theta, U, q, \lambda, \gamma) = (\theta v(q(\theta)) - cq(\theta) - U)f(\theta) + \lambda v(q(\theta)) + \gamma(U + \delta U^+ - \theta v(q(\theta))), \quad (57)$$

where λ is the co-state variable for the incentive-compabilitity constraint and γ is the multiplier for the limited enforcement constraint. Noting that the necessary conditions are also sufficient (Seierstad and Sydsaeter, 1986) (pg. 276). The relevant optimality conditions are:

$$f(\theta)[\theta v'(q(\theta)) - c] + \lambda(\theta)v'(q(\theta)) = \gamma(\theta)\theta v'(q(\theta))$$

and

$$\dot{\lambda}(\theta) = f(\theta) - \gamma(\theta)$$

which imply

$$\gamma(\theta) = f(\theta) - \frac{cf(\theta)}{\theta v'(q(\theta))} + \frac{F(\theta) - \Gamma(\theta)}{\theta}.$$

Therefore, a higher level of quantity $q(\theta)$ is implies with a lower $\gamma(\theta)$.

For $\gamma_\tau(\theta) = 0$ for some finite $\tau > \tau^*$ for all θ. Suppose otherwise, such that $\gamma_\tau(\tilde{\theta}) > 0$ for some $\tilde{\theta}$ and all τ. Then, $\Gamma_\tau(\theta) < 1$ for all $\theta \leq \tilde{\theta}$. Therefore, $1 - \Gamma_\tau(\theta) > 0$ for all $\theta \leq \tilde{\theta}$. Thus, as $\tau \to \infty$, $\sum_{s=0}^{\tau}(1 - \Gamma_s(\theta)) \to \infty$ for all $\theta \leq \tilde{\theta}$. Thus, as long as $q_\tau(\theta) < \infty$ for all θ, τ, it must be the case that some finite τ^* exists such that $\gamma_\tau(\theta) = 0$ for all $\tau > \tau^*$ and for all θ.

For $q_{\tau^*}(\theta) > q_\tau(\theta)$ for all $\tau < \tau^*$ and all θ. Notice that $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ if and only if

$$\theta\gamma_\tau(\theta) + \sum_{s=\tau+1}^{\tau^*-1}(1 - \Gamma_s(\theta)) \geq 0,$$

which always holds. It holds with strict inequality whenever the enforcement constraint binds, or when it binds in some period between τ and τ^* for some θ between θ and $\overline{\theta}$.

□

Proof of Proposition 3. Use the marginal price function $T'_\tau(q) = \theta_\tau(q)v'(q)$. Average unit prices $p_\tau(q)$ for $q > 0$ are given by:

$$p_\tau(q) = \frac{T_\tau(q)}{q} = \frac{\int_0^q \theta_\tau(x)v'(x)dx}{q},$$

where I have used the normalization $T_\tau(0) = 0$ and the inverse function $\theta_\tau(q)$. Average prices decrease over time if and only if

$$\int_0^q \theta_\tau(x)v'(x)dx > \int_0^q \theta_{\tau+1}(x)v'(x)dx$$

$$\Longleftrightarrow$$

$$\int_0^q [\theta_\tau(x) - \theta_{\tau+1}]v'(x)dx > 0.$$

By assumption, $q_\tau(\theta) \geq q_{\tau+1}(\theta)$ (and strictly so for $\underline{\theta}$). Thus, $\theta_\tau(q) > \theta_{\tau+1}(q)$ for all q and the inequality holds. □

SM4 A Two-Type Illustrative Example

The purpose of this example is four-fold. First, I illustrate how the introduction of the limited enforcement constraint may distort quantities relative to perfect enforcement. Second, I show that lower types unambiguously reap higher net returns due to the enforcement constraint. The introduction of the enforcement constraints effectively raises their reservation return to participate in trade, forcing the seller to offer larger shares of surplus to lower types. Third, I demonstrate that the optimal contract must be non-stationary. Fourth, I show through a solved example that the optimal stationary contract features *backloading*: unit prices decrease while quantities increase as relationships age.

SM4.1 Buyer's Types

A buyer type-θ gains a gross return θq^β from q units of the product sold by the seller. Assume there are positive, yet diminishing marginal returns, i.e., $\beta \in (0,1)$. The buyer types can take values $\{\theta_L, \theta_H\}$, such that $\theta_L < \theta_H$. Let f_L (resp. f_H) be the probability that buyer is type L (resp. type H) and assume no exit, i.e., $X(\theta) = 0$.

SM4.2 A Stationary Contract

For now, consider the optimal *stationary* contract. The optimal choice gives the buyer the net return $R(\theta_i) = \theta_i q_i^\beta - T(q_i)$. The seller designs the scheme to maximize:

$$max_{\{T_i,q_i\}} f_L(T_L - cq_L) + (1 - f_L)(T_H - cq_H)$$

where $T_i \equiv T(q_i)$, subject to incentive-compatibility constraints:

$$R(\theta_H) \equiv \theta_H q_H^\beta - T_H \geq \theta_H q_L^\beta - T_L, \tag{IC-H}$$

$$R(\theta_L) \equiv \theta_L q_L^\beta - T_L \geq \theta_L q_H^\beta - T_H. \tag{IC-L}$$

as well as the limited enforcement constraint:

$$\frac{\delta}{1-\delta}(R(\theta_i)) \geq T_i \quad i = L, H. \tag{LE-i}$$

This last constraint effectively (weakly) raises the minimum net rent that each buyer needs to obtain to participate in trade. The usual nonlinear pricing problem only requires that $R(\theta_i) \geq 0$. Instead, the limited enforcement case requires that $R(\theta_i) \geq (1-\delta)/\delta T_i > 0$, where the minimum return is endogenously determined. Notice that as $\delta \to 1$, the limiting case becomes the standard nonlinear pricing problem.[37]

To simplify the problem, assume that the IC-L and LE-H are slack while IC-H and LE-L are binding.[38] By using these assumptions on the constraints, one can obtain the optimal quantity allocations:

$$q_H^* = \left(\frac{\beta}{c}\theta_H\right)^{\frac{1}{1-\beta}},$$
$$q_L^* = \left(\frac{\beta}{c}\left[\theta_L - \frac{(1-\delta)\theta_L}{f_L} - \frac{(1-f_L)(\theta_H - \theta_L)}{f_L}\right]\right)^{\frac{1}{1-\beta}},$$

[37] The theoretical result that the buyer benefits from a deterioration of enforcement was previously discussed by Genicot and Ray (2006). In their model, they find that if better enforcement brings with it the deterioration of outside options and the seller has the bargaining power, the buyer will see their expected payoff increase. The opposite holds when the buyer has the bargaining power.

[38] All slack constraints are verified for the numerical example discussed below.

and optimal transfers:

$$\begin{aligned}T_H^* &= \theta_H q_H^\beta + (\delta\theta_L - \theta_H) q_L^\beta,\\ T_L^* &= \delta\theta_L q_L^\beta.\end{aligned}$$

The program's solution implies there is no distortion in quantities for type-H, as they purchase at the first-best level. However, type-L's purchases are shifted downwards. First, as is common in adverse selection problems, their purchases are distorted downwards to incentivize the revelation of type-H.

Second, contrary to the standard problem, extracting all rents from type-L is no longer feasible. As such, the standard quantity allocation for θ_L (i.e., when $\delta = 1$), together with the optimal transfers for L under limited enforcement do not satisfy IC-H. To see this, notice that as IC-H was binding in the standard problem, type-H was on the margin between their standard bundle and the standard bundle for type-L. Thus, if the limited enforcement bundle for type-L keeps quantities fixed (relative to the standard menu) and at the same time asks or lower transfers, type-H buyers would now prefer the menu intended for type-L. As a result, the seller needs to reduce type-L's allocation, even further than would be required under the standard adverse selection problem.

SM4.3 Non-Stationarity

Relative to the standard problem, the seller now needs to offer positive net returns to all buyers, in order to prevent default. Contrary to the results in Baron and Besanko (1984), the stationary contract is no longer the optimal contract. Instead, the seller could offer a dynamic contract with intertemporal incentives that uses the promise of future returns to the buyer to discipline their behavior now. Through this approach, the seller can extract higher shares of surplus early on than would be feasible under a stationary contract, increasing their present-value lifetime profits.

The exact dynamic path depends on the return function and distribution of types of the buyer, as well as the marginal cost of the seller and the common discount factor. For that reason, I consider next a solved numerical example.

SM4.4 A Visual Example

To visualize the problem, I consider a numerical example with the following values for the parameters: $\beta = 0.25$, $c = 1$, $f_L = 0.95$, $\theta_L = 10$, $\theta_H = 20$, $\delta = 0.9$.

Supplemental Material Figure SM6 shows the levels of quantities, prices, profits per buyer, and buyer's net return for the example discussed above for different regimes: stationary with perfect enforcement, stationary with limited enforcement, and dynamic with limited enforcement.

With the solid lines, the figure shows the stationary solution both under weak enforcement and perfect enforcement. In solid green, the figure shows the allocation for type-H. As mentioned above, limited enforcement of contracts does not distort their consumption relative to perfect enforcement. In solid blue, the figure shows the allocation for type-L under perfect enforcement. Type-L receives lower quantities and higher prices than type-H and receives zero net return.

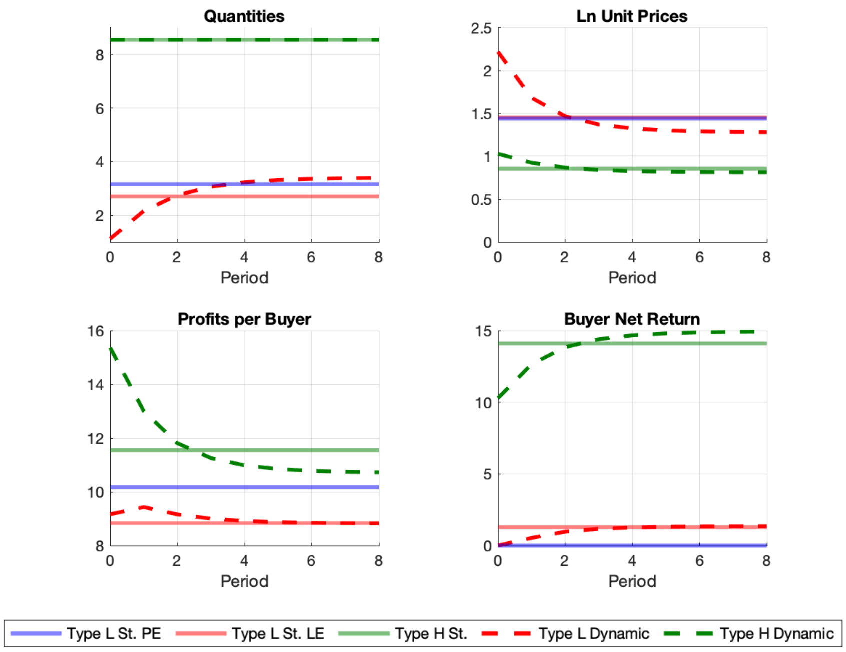
In solid red, the figure shows the allocation for type-L under limited enforcement. Relative to perfect enforcement, type-L sees a reduction in quantities and an increase in net return, in line with the logic explained above. Importantly, as the buyer's return function features diminishing returns in q, lower levels of quantity for lower values of δ also imply the seller can charge *higher* unit prices to type-L.

Lastly, the figure shows the optimal non-stationary path of prices and quantities in the dashed lines. The optimal path features *backloading* as quantities (weakly) increase and unit prices (weakly) decrease over time. As shown in the figure, this path of prices and quantities increases expected present-value lifetime profits from each buyer relative to the optimal stationary contract. The seller can effectively prevent default now and increase present-value lifetime profits by offering higher surplus levels to the buyers in the future.

Interestingly, the optimal path in the solved example features consumption for type-L in the long-run that is greater than the stationary contracts with and without limited enforcement. That is, through dynamic contracts, long-term allocations could potentially be more efficient than contracts under perfect enforcement.

In any case, the example shows that through the interaction market power on the seller side (which is reflected in the ability to offer incentive-compatible profit-maximizing menus) and the limited enforcement constraint, long-term contracts may display dynamics in which average quantities increase and unit prices decrease over time. Moreover, at any point in time, types consuming higher levels of quantities also enjoy lower unit prices. That is, this model of price discrimination with limited enforcement of contracts features i) *backloading* of prices and quantities, and ii) *quantity discounts* at any point in time.

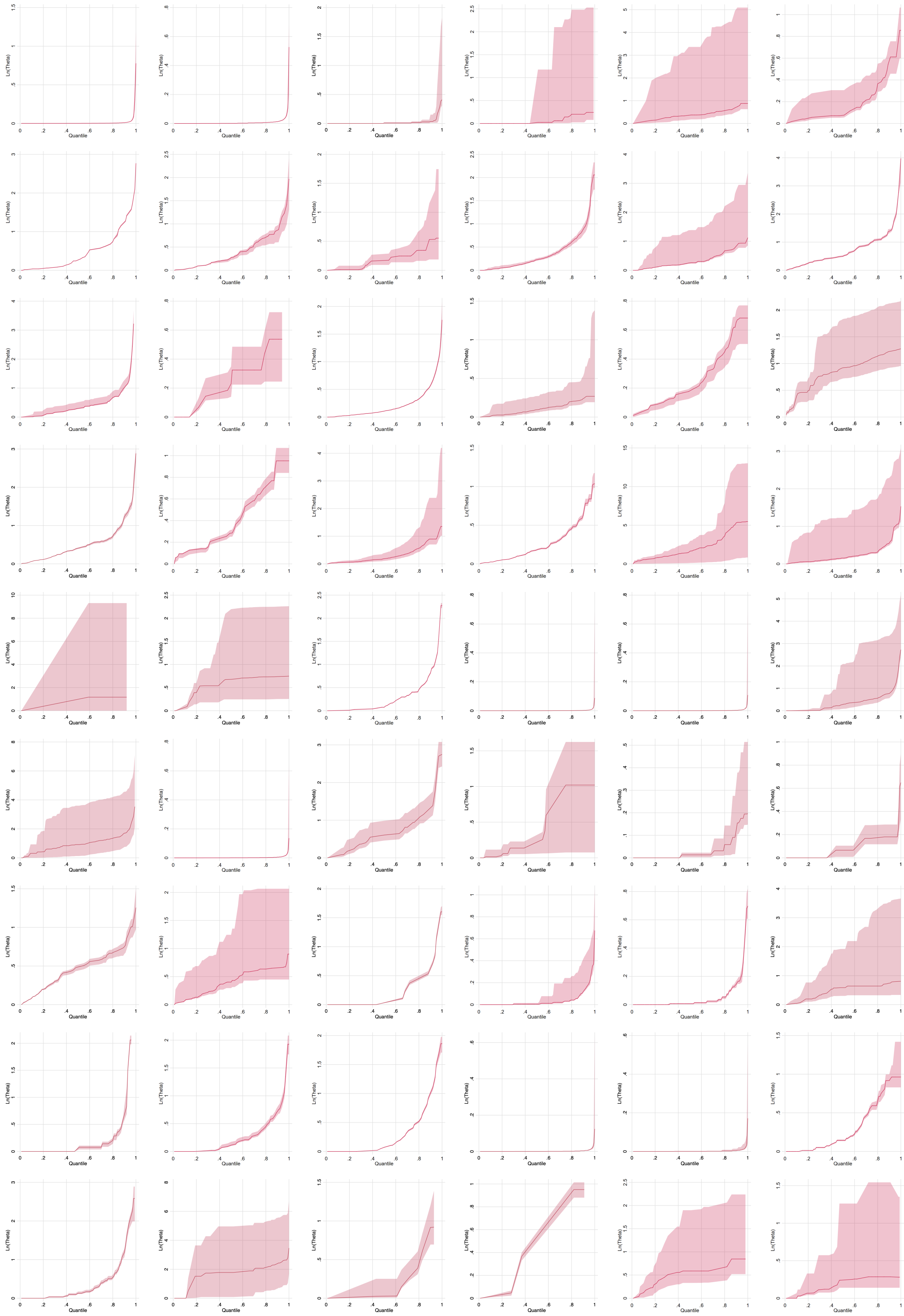
Figure SM6: **Example - Nonlinear Pricing and Limited Enforcement**

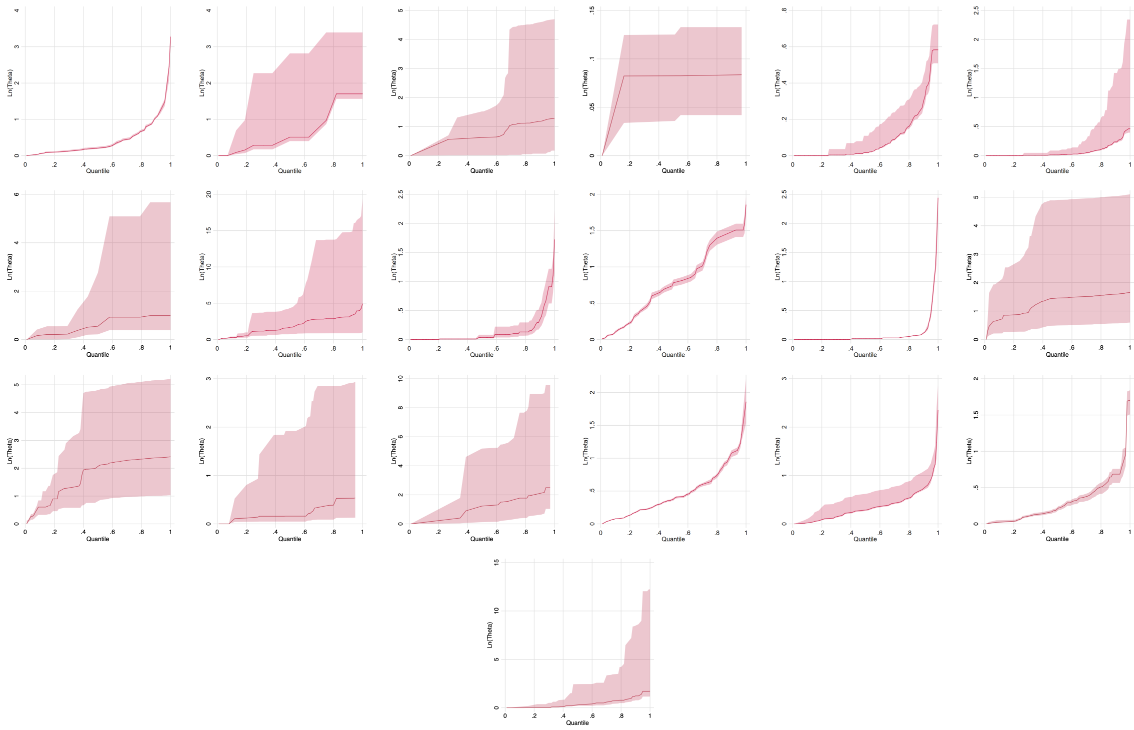


Notes: This figure shows Quantities, Prices, Profits, and Buyer Net Return for different enforcement and contract regimes. In solid green, the optimal stationary contract for type-H under perfect enforcement and limited enforcement. In dashed green, the optimal dynamic contract for type-H under limited enforcement. In solid blue, the optimal stationary contract for type-L under perfect enforcement. In solid red, the optimal stationary contract for type-L under limited enforcement. In dashed red, the optimal dynamic contract for type-H under limited enforcement. The parameters used in the example are: $\{\beta = 0.25,\ c = 1,\ f_L = 0.95,\ \theta_L = 10,\ \theta_H = 20,\ \delta = 0.9\}$.

SM5 Bootstrapped Distribution of Types

In this section, I present the boostrapped distribution for types of each seller-year.




Ln(Theta)
Quantile

Supplemental Material References

Baron, D. P. and Besanko, D. (1984). Regulation and information in a continuing relationship. *Information Economics and Policy*, 1(3):267–302.

Genicot, G. and Ray, D. (2006). Bargaining power and enforcement in credit markets. *Journal of Development Economics*, 79(2):398–412.

Seierstad, A. and Sydsaeter, K. (1986). *Optimal control theory with economic applications.* Elsevier North-Holland, Inc.