Online Appendix

OA-1 Data Construction and Summary Statistics

OA-1.1 Data Construction

The study defines a *product* as a bar-code identifier and description combination. While discounts are observed at the product level, I allocate the discounts offered in a transaction equally across all the products purchased in that transaction by adjusting the listed product unit prices. For example, if a 5% discount is offered on the total bill, the reported unit prices of all the products are adjusted by 5%. I do this, rather than performing analysis with observed discounts to average out managerial mistakes, such as assigning all discounts to a single product, while in principle the agreed discounts were on the total bill.[43]

Let p^l_{ijgry} be the listed unit price and q_{ijgry} be the reported quantity for buyer i from seller j for good g in transaction r during year y, and d_{ijry} be the total discount share in the transaction. Then, the effective unit price is defined as $p_{ijgry} = (1 - d_{ijry}) \times p^l_{ijgry}$. Following DellaVigna and Gentzkow (2019), I define standardized unit prices at the transaction-product level $\tilde{p}_{ijgry}$ as:

$$\tilde{p}_{ijgry} = ln(p_{ijgry}) - \overline{ln(p_{jgy})}, \tag{34}$$

where $\overline{ln(p_{jgy})}$ is the average log effective unit price for the good g of seller j in year y. The standardized unit price captures the percentage price difference for a given product in a transaction relative to its average yearly price. I define standardized quantity at the transaction-product level $\tilde{q}_{ijgry}$ in an analogous manner:

$$\tilde{q}_{ijgry} = ln(q_{ijgry}) - \overline{ln(q_{jgy})}, \tag{35}$$

where $\overline{ln(q_{jgy})}$ is the average log quantity for the good g of seller j in year y. As with prices, standardized quantities measure the percentage quantity difference for a given product in a transaction relative to its average quantity sold in the year. Note that these definitions for standardized units are equivalent to netting out product-seller-year fixed effects in a regression of log effective unit prices or log quantities.

To obtain pair-year-level values of the standardized prices and quantities, I aggregate them by the respective share of total expenditures, which provides a common weight for prices and quantities. Define V_{ijy} as the total value of transactions between buyer i and seller j in year y. Let $s_{ijgry} = v_{ijgry}/V_{ijy}$ be the share of expenditure that good g in transaction r represents for the pair and $v_{ijgry} = p_{ijgry} * q_{ijgry}$ be the transaction value.[44] Then, define pair-year level

[43] An alternative method would be to use observed discount shares at the product level and adjust listed prices by the product-specific discount share. In practice, the reduced-form facts hold using either method. See, for instance, Online Appendix Table OA-7 for a robustness exercise using product-level vs bill-level allocation of discounts.

[44] Again, reduced-form results are robust to relying on quantities as weights, rather than values (Online Appendix Table OA-7).

equivalents for the standardized prices and quantities as:

$$\tilde{p}_{ijy} = \sum_{r\in R_{ijy}} \sum_{g\in G_{ijry}} s_{ijgry} * \tilde{p}_{ijgry}, \qquad (36)$$
$$\tilde{q}_{ijy} = \sum_{r\in R_{ijy}} \sum_{g\in G_{ijry}} s_{ijgry} * \tilde{q}_{ijgry},$$

where R_{ijy} is the set of all the transactions between i and j in year y and G_{ijry} is the set of all goods in transaction r. The pair-level standardized price then captures the average relative price a buyer has in a given year. For instance, if $\tilde{p}_{ijy} = 0.1$, then the buyer pays on average 10% on their products than other buyers. The pair-level quantities capture the average relative quantity a buyer purchases in a given year. Thus, if $\tilde{q}_{ijy} = 0.1$, then the buyer purchases 10% more in quantity than other buyers.

To address the potential concern that cross-sectional differences in prices and quantities could be driven by variations in the bundles of goods purchased by buyers and over time, I report the main stylized facts on the patterns and dynamics of prices and quantities using standardized measures. The use of these measures indicates that differences in the products purchased by buyers do not influence the results.

For estimation purposes, however, I use the following definitions of prices and quantities, as they are better suited to the structure of the model. For total quantity q_{ijy}, I sum over all reported quantities over all goods and all transactions:

$$q_{ijy} = \sum_{r\in R_{ijry}} \sum_{g\in G_{ijry}} q_{ijgry}. \qquad (37)$$

As discussed below in this Section, aggregation across products is not extremely problematic, as firms tend to produce either items or packages that can be summed over in a relatively consistent way.

For prices, I obtain the average unit price by dividing the total value of transactions by the total quantity:

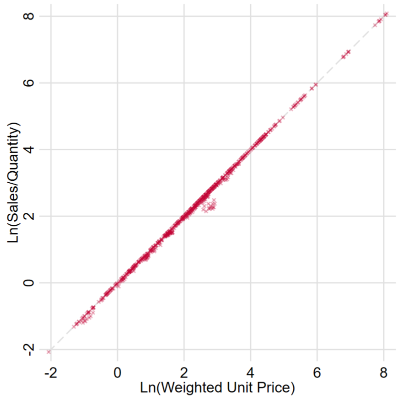
$$p_{ijy} = V_{ijy}/q_{ijy}. \qquad (38)$$

This definition of prices is consistent with the weighted average of product-level effective prices, as demonstrated in Online Appendix Figure OA-1, which presents the fit between average unit prices and weighted effective unit prices.[45] The figure shows a strong fit between the two measures, with a correlation of 0.58 at the buyer-seller-year level.

The aggregate quantity produced by seller j in year y is given by $Q_{jy} = \sum_{i\in I_{jy}} q_{ijy}$, where I_{jy} is the set of all buyers that transacted with the seller in the year. While the measures of quantities differ between the model and the motivating evidence, all motivating facts hold when using total quantities, both in the cross-section and in the short panel structure (controlling for buyer-seller pair fixed effects). Robustness results using average unit prices and total quantity are also discussed below.

[45] Observed weighted prices are obtained by aggregating unit prices using the share of the total quantity of the goods sold as weights.

Figure OA-1: **Average Price vs Weighted Price**



Notes: This figure plots average unit prices (in logs) against weighted prices (in logs) across buyer-seller-year. Average unit prices are calculated by dividing the total value of yearly transactions by total quantity purchased (pooling different products). Weighted prices are calculated by summing transaction-product-level unit prices with total expenditure share as weights.

OA-1.2 Main Summary Statistics

Table OA-1 shows that the sellers in my sample are typically large and well-established, employ directly imported goods in their production, channel their sales through the local market rather than exporting. On the other hand, buyers are smaller, younger, and have limited direct contact with international trade. Moreover, buyers are less capital-intensive than sellers. At the same time, sellers in the same 6-digit industry but not in my sample are orders of magnitude smaller, younger, do not use imported inputs, and are much less capital-intensive than seller in sample.[46]

Table OA-1: **Summary Statistics - Sellers and Buyers in 2016**

	Sellers - Sample			*Buyers*			*Sellers - Not Sample*		
	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
Total Sales (million USD)	14.95	8.26	24.33	2.35	0.20	24.33	0.10	0.00	3.04
Total Inputs (millon USD)	10.58	5.31	18.94	1.92	0.15	24.13	0.07	0.00	1.86
Age	30.47	29.00	19.16	15.18	14.00	9.75	9.24	7.00	8.88
Import Share (%)	24.47	21.38	22.96	3.82	0.00	13.49	0.30	0.00	3.91
Export Share (%)	5.81	0.00	19.11	1.06	0.00	8.87	0.10	0.00	2.81
Capital-Expenditures Ratio	0.27	0.30	0.18	0.16	0.05	0.23	0.02	0.00	0.10
Observations	49			28,138			28,424		

Notes: This table reports summary statistics about the size, age, capital intensity, and trade exposure of buyers and sellers in the sample for the year 2016. Monetary values are in U.S. dollars for 2016.

Table OA-2 presents the industrial composition of buyers categorized by selling sector. Buyers of Textile products are primarily from the Wholesale and Retail sector, followed by Manufacturing. Pharmaceutical products are mostly purchased by entities in the Wholesale and Retail sector, as well as the Human Health sector, which includes hospitals and doctors. Cement products, on the other hand, are mainly bought by businesses in Wholesale and Retail, Construction, and Professional Services, such as engineering and architectural firms. Across all

[46] The large number of sellers not in sample is driven primarily by thousands of micro-entrepreneurs in textiles. Online Table Appendix Table OA-8 presents the sample descriptive statistics by seller industry.

selling sectors, the predominance of buyers in Wholesale and Retail Trade suggests that buyers likely have linear input needs.

Table OA-2: **Industrial Composition of Buyers by Selling Sector**

Seller Industry	Ranking	Buyer Industry	Average % Share Pairs
Textiles	1	Wholesale & Retail	40
Textiles	2	Manufacturing	15
Textiles	3	Professional Activities	8
Textiles	4	Agriculture	5
Textiles	5	Other	31
Pharmaceutical	1	Wholesale & Retail	46
Pharmaceutical	2	Human Health	17
Pharmaceutical	3	Manufacturing	10
Pharmaceutical	4	Construction	4
Pharmaceutical	5	Other	23
Cement-Products	1	Wholesale & Retail	25
Cement-Products	2	Construction	20
Cement-Products	3	Professional Activities	16
Cement-Products	4	Manufacturing	8
Cement-Products	5	Other	31

Notes: This table presents a ranked breakdown of the industrial composition of buyers for each selling sector, organized by the highest share of buyers.

Table OA-3 presents summary statistics on quantities, values, and the number of buyers per seller obtained through the EI dataset. Notice that the reporting threshold is smaller than in previous work (Bernard et al., 2022; Alfaro-Urena et al., 2022), implying a larger number of buyers. Despite the large number of buyers, the yearly bills are not small for the country, with median (average) bill of 9K USD (44K USD).[47]

Table OA-3: **Summary Statistics - Electronic Invoice Database**

	Mean	Median	SD
N. Buyers	8,028.41	613.50	25,078.11
Total Sales (million USD)	16.58	7.23	29.44
Total Q (million)	5.42	1.20	9.01
Q per Buyer	12,455.39	1,495.22	25,823.40
Bill per Buyer (USD)	43,490.37	9,067.65	105,840.28
Observations	49		

Notes: This table reports summary statistics of the electronic invoice database. N. buyers refers to the number of unique buyers each seller in the sample has on average over 2016 and 2017. Quantity is the sum of all quantities across products. Bill per buyer is the total value of the transactions between buyer and seller.

The median (average) buyer purchases around 1.5K (12.5K) units of product. What are these products? Table OA-4 provides information on a random sample of products, including

[47] At the same time, due to the staggered rollout of the policy, data is sourced from the largest firms in the economy. Indeed, the size in number of buyers and total sales of the median manufacturing firm in my sample corresponds to size of manufacturing firms between the top 5 and 10 percent in Costa Rica (Alfaro-Urena et al., 2022) and between the top 25 and 10 percent in Belgium (Bernard et al., 2022).

their prices and average costs. The prices are obtained directly from invoices, while the average costs are imputed by dividing the total variable costs, including wages and intermediate inputs, by nthe aggregate output in units for each firm.

Table OA-4: **Example - Product Information, Prices, and Average Costs**

Industry	Firm-ID	Product Description	Observed Unit Price	Imputed Average Cost
Textiles	1	Teddy King, Size 55, Brim 7CM, Color-B02 [Panama Hat]	33.90	11.96
Textiles	2	Shirt, R:1931, Squares	19.34	9.85
Textiles	3	Tank Undershirt, Male, Size M, White	10.27	6.72
Textiles	4	Betty K246	19.44	16.94
Textiles	5	Bikini, Woman, 500306, Black, L	13.50	16.78
Textiles	6	Ribbon, Black, 30 mm X 700	26.62	1.86
Textiles	7	Skirt, Tropical Squares, Scottish	46.01	17.77
Textiles	8	Boots, LLN NG AM, Size 39	7.09	2.17
Textiles	9	Elastic Socks, Nylon and Cotton	16.56	8.48
Textiles	10	Jacket, Kids, Spiderman Print, Hoodie	18.30	7.11
Pharmaceutical	1	Nitazoxanida, 500mg X 6 tablets	5.27	4.83
Pharmaceutical	1	Clopidogrel Tarbis 75 mg film-coated tablets	12.90	6.57
Pharmaceutical	2	Losartan/Hydrochlorothiazide, 100mg X 28 tablets	5.04	0.78
Pharmaceutical	3	B Complex, Syrup 120 ml	2.32	0.81
Pharmaceutical	4	Sodium perborate, mint oil, saccharin	4.69	1.81
Pharmaceutical	5	Boldenone 50, Injectable, Bottle X 500 ml	123.12	3.01
Pharmaceutical	6	Pinaver, Film-coated, 100 mg X 20 tables	10.32	2.62
Pharmaceutical	7	Endobion X 60 tablets	14.83	5.49
Pharmaceutical	7	Prostageron X 60 capsules	14.75	7.04
Pharmaceutical	8	Oral rehydration solution, cherry, 500ml	2.67	1.80
Cement-Products	1	Gray French Pedestrian Paving Stone	11.28	18.11
Cement-Products	2	Corrugated Plate	23.73	9.56
Cement-Products	3	Polymer-modified adhesive mortar for ceramics, 25kg	6.31	2.99
Cement-Products	4	Polymer-modified adhesive mortar for ceramics, 25kg	6.94	12.36
Cement-Products	5	Polymer-modified adhesive mortar for ceramics, 25kg	6.65	3.45
Cement-Products	6	Straight Pole 21m x 1400kg, Reinforced Concrete	882.00	73.95
Cement-Products	6	Straight Pole 21m x 2400kg, Reinforced Concrete	1362.73	73.95
Cement-Products	7	Tile 50x50x2 cm (Color)	32.00	6.62
Cement-Products	8	MFC Concrete, 300, XXXXX XXXX-XXXX	94.00	50.34
Cement-Products	8	CFC Concrete, 240, XXXXX XXXX-XXXX	79.43	50.34

Notes: This table presents a sample of ten random products from each of the studied sectors (textiles, pharmaceutical, and cement-products), with product descriptions translated into English and sensitive information, such as brand names, removed to ensure confidentiality. The observed average unit prices reflect the listed prices reported by the firms, while the imputed average costs are estimated using the firms' total variable costs divided by total quantity.

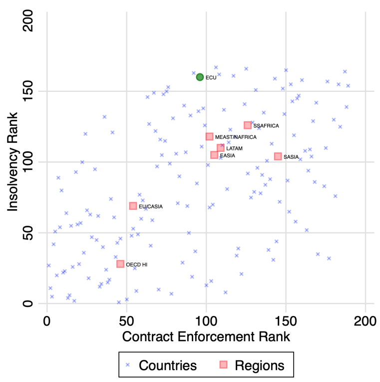
In the textiles industry, products may include shirts, skirts, hats, and others, with different patterns or sizes also considered separate products. Aggregation is thus over individual clothing *items*. Instead, in the pharmaceutical sector, products are typically packages of tablets or bottles, with aggregation across products being over *packages*. Comparing product-level prices with firm-level average costs yields reasonable estimates in both cases. For example, a shirt is sold for 19 USD and costs 9.85 USD to manufacture, and Vitamin B Syrup is sold for 2.3 USD, but it costs only 81 cents to manufacture.

In the cement-products industry, products may include stones, mortar, concrete, and the like. While aggregating over these types of products can be more challenging, it should be noted that firms producing products such as mortar do not typically produce tiles, poles, or stones. Two other notes are in order. First, there are three different firms selling mortar at similar prices, despite being headquartered in different and distant cities. This suggests that despite the products being substitutes, sellers may still have local market power due to transportation costs. Second, one firm produces two types of pole products, sold at different prices but with the same cost of production. Another firm produces two types of concrete products, sold at different prices but with the same cost of production. As costs will enter into the dependent variable in my main estimation process, possible mistakes in costs would enter as measurement error in the econometric model.

OA-1.3 Context Related Summary Statistics

Online Appendix Figure OA-2 shows Ecuador's position in terms of contract enforcement and insolvency in the World Bank Doing Business report. Lower numbers represent better institutions to enforce contracts or resolve insolvency cases.

Figure OA-2: **Ranks Insolvency and Enforcement**



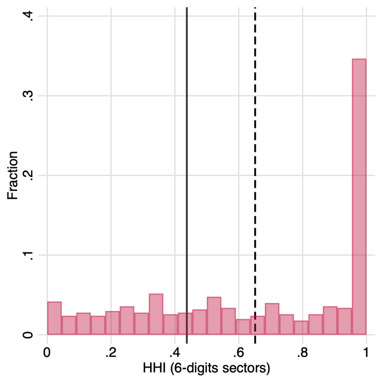
Notes: This figure presents Ecuador's rank in the World Bank Doing Business categories of Insolvency (Y-Axis) and Enforcement (X-Axis). The most efficient country in terms of enforcement ranks 1st.

Online Appendix Figure OA-3 shows the distribution of Herfindahl-Hirschman Indices (HHI) for 6-digit manufacturing sectors in 2017. HHI_s for sector s is estimated using the following formula:

$$HHI_s = \sum_{j \in J_s} m_j^2,$$

where m_j is the market share of firm j, and J_s is the set of active firms in sector s. The market share of firm j is obtained by dividing the total revenue of firm j by the total revenue of all firms in sector s.

Figure OA-3: **Distribution of Herfindahl-Hirschman Indices for Manufacturing in 2017**



Notes: This figure presents a histogram of estimated Herfindahl-Hirschman Indices (HHI) for 6-digit manufacturing sectors in 2017.

OA-2 Motivating Evidence - Robustness

Quantity Dynamics and Pair FE. In Online Appendix Figure OA-4a, I verify that the differences are not driven by selection, but rather reflect a real increase within pairs. To do so, I run a regression of total quantity q_{ijt} on dummies for the age of the relationship, controlling for pair fixed effects. The figure plots the coefficients for the relationship age dummies and shows that the volume of total quantity purchased grows as relationships age.

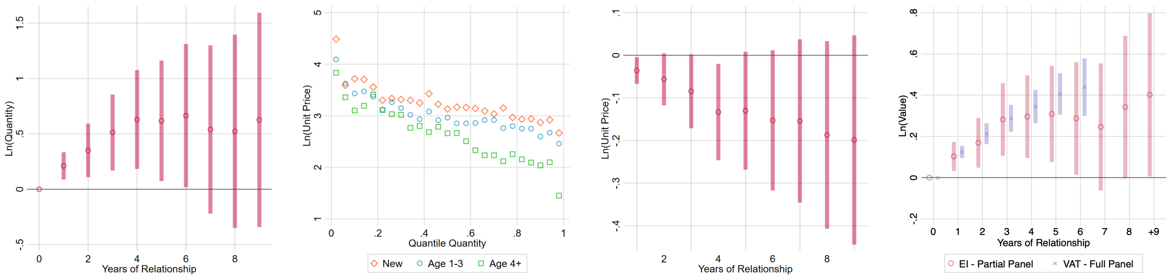
Quantity Discounts and Pair FE. Online Appendix Figure OA-4b plots a binscatter regression of log average unit price on quantiles of quantity, controlling for seller-year fixed effects. The figure documents the presence of quantity discounts within relationship age. *Relationship Discounts and Pair FE.* Online Appendix Figure OA-4c shows a binscatter plot of log average prices on the age of the relationship, controlling for pair fixed effects. The figure shows that as relationships age, they receive around 1.5% additional discounts per year. Under both formulations, there are price discounts conceded to older clients. These results indicate that the discounts are not driven by composition, nor by short-term fixed characteristics of the firm (such as location, managerial bargaining, size, etc.).

Relationship Value and Pair FE. Online Appendix Figure OA-4d plots regression coefficients for the value of total sales between buyer and supplier on the age of the relationship, controlling for pair fixed effects. The red figures use the electronic invoice database and are constructed using only a partial panel of two observations per pair for the years 2016-2017. The purple marks are constructed using multiple observations of buyer-seller pairs from the VAT B2B database for the years 2007-2015 for the sellers in the electronic invoice database. The figure confirms that relying on only two years of relationship data can properly capture full relationship dynamics observed in longer panel datasets.

Backloading in Prices and Quantities by Quantile and Relationship Age. Online Appendix Figure OA-5 presents pair-specific changes in prices and quantities between 2016 and 2017, by the age of the relationship in 2016, over quantiles of quantity purchased in 2016. The figures show that prices tend to decrease faster and quantities increase faster for lower quantiles. Over time, backloading in prices and quantities becomes weaker. By age 5, prices and quantities are relatively stable across quantiles.

Benchmarking Quantity Discounts. Online Appendix Table OA-5 shows the results of a regres-

Figure OA-4: **Motivating Facts - Robustness (Pair Fixed Effects)**



(a) Q. over Time (b) Q. Discounts (c) Prices over Time (d) Value over Time

Notes: Panel (a) plots the coefficients of log total quantity on relationship age dummies, controlling for pair fixed effects. Total quantity is obtained by aggregating all the reported units of sold goods. Standard errors are clustered at the pair level. Panel (b) shows the relationship between quantity purchased and average log unit price through binscatters of the measure of unit price against quantile of quantity by age of relationship. Quantiles of quantity are calculated for each seller-relationship age combination. Panel (c) plots the regression coefficients of log unit prices on years of relationship, controlling for pair fixed effects. Standard errors are clustered at the pair level. Panel d) plots regression coefficients for the value of total sales between buyer and supplier on the age of the relationship, controlling for pair fixed effects. The red figures use the electronic invoice database and are constructed using only a partial panel of two observations per pair for the years 2016-2017. The purple marks are constructed using multiple observations of buyer-seller pairs from the VAT B2B database for the years 2007-2015 for the sellers in the electronic invoice database.

sion on log average price on log quantity, controlling for seller-year fixed effects. The table presents a benchmark quantity discount measure of a 2% decrease in price for a 10% increase in total quantity purchased.

Table OA-5: **Benchmark: Quantity Discounts**

	(1)
VARIABLES	ln(Price)
ln(Quantity)	-0.220***
	(0.0238)
Constant	3.046***
	(0.0718)
Seller-Year FE	Yes
Observations	76,473
R-squared	0.666

Notes: This table presents a regression of log average unit prices on log quantity, controlling for seller-year fixed effects. Standard errors are clustered at the seller-year level. *** p<0.01, ** p<0.05, * p<0.1

Relationship Discounts and Additional Controls. I replicate Figure 1e in Table OA-6 to assess the robustness of the results to additional buyer and relationship-level controls, obtained from the firms' financial statements. Relative to the base specification presented in Column (1), I find that the effects of relationship age and quantity discounts remain relatively unchanged after accounting for various buyer and pair characteristics.

In Column (2), I control for buyer and pair characteristics such as age, distance between headquarters, size (measured by sales, number of employees, and assets), and whether the firm is a multinational, exporter, importer, or part of a business group. I also consider the impor-

Figure OA-5: **Backloading in Prices and Quantities by Quantile and Age**

(a) Prices (New Buyers)

(b) Quantities (New Buyers)

(c) Prices (Age 1-4)

(d) Quantities (Age 1-4)

(e) Prices (Age 5+)

(f) Quantities (Age 5+)

Notes: This figure presents pair-specific year-to-year changes in unit prices and quantities from 2016 and 2017 for new buyers, ages 1 to 4, and age 5+, against the quantile of quantity purchased in 2016. The age of relationships is from 2016, and quantiles of quantity are measured in 2016 for each seller-relationship age. Error bars present variation at the 95% level, with standard errors clustered at the seller level.

tance of the relationship for both the buyer (in terms of supply share) and seller (in terms of demand share) to capture any potential asymmetries in bilateral bargaining power (Dhyne et al., 2022; Alviarez et al., 2023). In Column (3), I include further controls, such as buyer wages, expenditures, cash, fixed assets, debt, leverage, and export and import shares, as well as 6-digit sectoral fixed effects for buyers. The stability of the coefficients implies that buyer characteristics observed by the seller, but not accounted for in a model focusing solely on relationship and quantity variation, likely enter as measurement error rather than generating bias in the coefficients linking prices, quantities, and relationship age.

In Columns (4) and (5), I substitute the relationship age with its logarithmic form, rather than in levels, and again find robust results for both discounts over time and by quantity. Importantly, prices are most responsive to quantities and the age of the relationship. For instance, the coefficient for the (log) age of the relationship is 4 to 10 times larger than the coefficient for the (log) age of the buyer, and 15 to 20 times larger than the coefficient for the (log) sales of the buyer.

Relationship Discounts and Aggregation Weights. Online Appendix Table OA-7 presents the robustness of relationship discounts to the method of discount allocation (at the bill level vs. at the product level) as well as the weights used to aggregate prices at the seller-buyer-year level (quantities vs. values as weights).

Table OA-6: **Standardized Log Price - Robustness to Additional Controls**

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Std. ln(Price)	Stdz. ln(Price)	Stdz. ln(Price)	Stdz. ln(Price)	Stdz. ln(Price)
Age of Relationship	-0.00554***	-0.00552***	-0.00480***		
	(0.00156)	(0.00146)	(0.00140)		
ln(Age of Relationship+1)				-0.0186***	-0.0161***
				(0.00500)	(0.00483)
Stdz. ln(Quantity)	-0.0472***	-0.0463***	-0.0420***	-0.0463***	-0.0420***
	(0.00780)	(0.00722)	(0.00414)	(0.00719)	(0.00414)
Supply Share		0.0262*	0.0183	0.0268	0.0184
		(0.0157)	(0.0137)	(0.0163)	(0.0148)
Demand Share		0.0119	0.0378	-0.00200	0.0250
		(0.0486)	(0.0400)	(0.0479)	(0.0388)
ln(Age Buyer)		-0.000836	-0.00368***	-0.00169	-0.00439***
		(0.00117)	(0.00117)	(0.00114)	(0.00124)
ln(Distance Km)		2.96e-05	0.000472	-2.77e-05	0.000424
		(0.00192)	(0.00194)	(0.00192)	(0.00194)
ln(Sales Buyer)		0.00108**	0.000774**	0.00110**	0.000804**
		(0.000469)	(0.000318)	(0.000477)	(0.000318)
ln(N. Employees Buyer)		0.000235	0.00161*	0.000256	0.00160*
		(0.000846)	(0.000932)	(0.000841)	(0.000936)
ln(Assets Buyer)		0.00131***	0.00228***	0.00132***	0.00230***
		(0.000318)	(0.000696)	(0.000319)	(0.000668)
ln(Wages Buyer)			-0.000472		-0.000471
			(0.000319)		(0.000322)
ln(Expenditures Buyer)			-0.000949**		-0.000926***
			(0.000377)		(0.000347)
ln(Cash Buyer)			0.000785**		0.000784**
			(0.000338)		(0.000335)
ln(Fixed Assets Buyer)			0.000507**		0.000501**
			(0.000253)		(0.000247)
ln(Debt Buyer)			-0.000456		-0.000453
			(0.000411)		(0.000410)
Leverage Buyer			0.000833		0.000835
			(0.00195)		(0.00194)
1{BG Buyer}		-0.00374	7.80e-05	-0.00383	-7.23e-06
		(0.00236)	(0.00207)	(0.00236)	(0.00207)
1{Multinational Buyer}		0.0194		0.0197*	
		(0.0120)		(0.0118)	
1{Exporter Buyer}		-0.00747	-0.0239**	-0.00721	-0.0239**
		(0.00566)	(0.00937)	(0.00553)	(0.00948)
Export Share Buyer			0.0321***		0.0314***
			(0.00793)		(0.00791)
1{Importer Buyer}		0.00369**	0.000511	0.00357*	-0.000107
		(0.00184)	(0.00440)	(0.00183)	(0.00442)
Import Share Buyer			0.0105*		0.0112*
			(0.00562)		(0.00579)
Observations	76,412	73,633	65,754	73,633	65,754
R-squared	0.075	0.082	0.048	0.083	0.048
Year FE	Yes	Yes	Yes	Yes	Yes
Buyer Sector FE	No	No	Yes	No	Yes

Notes: This table presents regressions regressions of standardized unit prices on age of relationship, standardized quantity, and different buyer characteristics. Standard errors are clustered at the seller-year level. *** p<0.01, ** p<0.05, * p<0.1

Table OA-7: **Robustness to Weights and Discount Allocation**

	(1)	(2)	(3)	(4)
Variable (Weighted Average)	Stdz. Price	Stdz. Price	Stdz. Price	Stdz. Price
Age of Relationship	-0.00690***	-0.00685***	-0.0106***	-0.00884**
	(0.00187)	(0.00177)	(0.00401)	(0.00433)
Weights	Values	Quantity	Values	Quantity
Discount Allocation	Bill	Bill	Product	Product
Observations	76,473	76,473	76,473	76,473
R-Squared	0.018	0.018	0.015	0.009
Pair FE	No	No	No	No
Year FE	Yes	Yes	Yes	Yes
Quantity Control	No	No	No	No

Notes: This table presents regressions of prices on the age of the relationship under different weights for aggregation and methods of allocating discounts. Column (1) is the benchmark and allocates discounts at the bill level, relying on the value share of total yearly transactions as aggregation weights. Column (2) allocates discounts at the bill level and uses total quantity as weights. Column (3) allocates discounts at the product level with values as weights. Column (4) allocates discounts at the product level with quantities as weights. Standard errors are clustered at the seller-year level. *** p<0.01, ** p<0.05, * p<0.1

OA-3 Motivating Facts by Seller Sector

In this section, I present the overall consistency of the motivating facts for each seller sector: namely, textile, cement-products, and pharmaceuticals.

Online Appendix Table OA-8 presents summary statistics by seller's sectors for *Sellers in Sample* (Panel (a)), *Sellers Not in Sample* (Panel (b)), which are sellers in the same industry but small enough that they were not covered by the EI seller's database, and *Buyers in Sample* (Panel (c)). The table demonstrates that the sellers in the sample are significantly larger than their non-sample competitors, with the mean sample seller being 272 times larger in the textile industry, 8 times larger in the pharmaceutical industry, and 32 times larger in the cement-products industry. Furthermore, the sample sellers exhibit a higher exposure to imported materials compared to their non-sample counterparts, with 113 times more reliance in textiles, 4 times more in pharmaceuticals, and 26 times more in cement-products. Additionally, the firms in the sample display a considerably higher capital intensity, with 18 times more capital per dollar in expenditure in textiles, 2 times more in pharmaceuticals, and 8 times more in cement-products. These patterns collectively suggest that (1) the manufacturing firms in the sample are preferred suppliers within their respective industries, (2) there is a degree of product differentiation, likely indicating higher quality given the increased reliance on imported inputs, and (3) the higher capital intensity relative to labor implies a reduced likelihood of production issues.

These statistics also help understand why sellers in the sample have market power in the first place. For textile-products: The average buyer in this industry is smaller than the sellers in the sample, yet significantly larger than the non-sample competitors (59 times larger). Furthermore, the average order in the industry, at 25,000 USD, is substantial relative to the size of the average (40,000 USD) and median ($<$ 9,000 USD) non-sample seller. Therefore, beyond the potential higher quality of goods offered by the sellers in the sample, the relatively large size requirements for the orders imply a scale advantage for in-sample sellers.

In the pharmaceutical-products industry: Products are generally horizontally differentiated, as active components are imperfect substitutes for the final consumer. The size, age, capital intensity, and reliance on imported inputs suggest that sellers in the sample are the preferred suppliers in this differentiated industry.

In the cement-products industry: Manufacturers likely benefit from local market power due to the high transportation costs associated with these types of goods. Additionally, the manufacturers in the sample are likely vertically differentiated due to their capital-intensive production. Similar to the textile industry, a scale argument is valid as well. The average buyer in the industry is 14 times larger than the average non-sample seller, and the orders are relatively large (45,000 USD) compared to the size of the non-sample seller (350,000 USD average; 10,000 USD median).

Decomposing Figure 1 by sector reveals that the vast majority of qualitative results hold individually in each sector, with Online Appendix Figure OA-6 for Textiles, Online Appendix Figure OA-7 for Pharmaceuticals, and Online Appendix Figure OA-8 for Cement-products.

First, a large share of trade is channeled through repeated relationships (Subfigure OA-6a; Subfigure OA-7a; Subfigure OA-8a), though pharmaceutical manufacturers have a lower share of new clients and quantity channeled through new buyers. Still, repeated transactions rather than spot transactions are thus likely important in each industry.

Second, at least 60% of all transactions are financed by trade-credit (Subfigure OA-6b; Subfigure OA-7b; Subfigure OA-8b). This implies that opportunism on the buyer side is relevant

Table OA-8: **Summary Statistics by Sector - Sellers, Buyers, and Other Competitors**

	Textiles			Pharmaceuticals			Cement-Products		
	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.
Panel (a): Sellers in Sample									
Total Sales (million USD)	10.91	3.55	21.26	21.64	12.18	31.78	11.32	7.10	12.89
Expenditures (million USD)	6.48	2.38	12.90	16.13	6.29	27.10	8.73	5.58	8.68
Age	31.47	29.00	18.55	30.89	33.00	20.73	28.25	22.00	19.17
Import Share (%)	20.28	11.67	21.85	35.94	27.65	24.11	13.92	4.99	15.94
Export Share (%)	14.04	0.21	29.20	1.00	0.00	2.14	0.01	0.00	0.05
Capital Share of Expenditures	0.18	0.13	0.18	0.28	0.32	0.15	0.39	0.44	0.16
Observations	19			18			12		
Panel (b): Sellers Not in Sample									
Total Sales (million USD)	0.04	0.00	0.69	2.81	0.04	11.54	0.35	0.01	7.51
Expenditures (million USD)	0.03	0.00	0.42	2.44	0.03	12.59	0.22	0.00	3.79
Age	8.94	6.00	8.75	14.32	10.50	14.17	10.83	9.00	8.99
Import Share (%)	0.18	0.00	2.90	9.94	0.00	21.89	0.53	0.00	5.01
Export Share (%)	0.09	0.00	2.67	1.37	0.00	8.90	0.13	0.00	2.89
Capital Share of Expenditures	0.01	0.00	0.08	0.17	0.06	0.24	0.05	0.00	0.17
Observations	24,320			234			3,870		
Panel (c): Buyers in Sample									
Total Sales (million USD)	2.37	0.18	25.66	6.21	0.31	58.74	5.01	0.55	44.49
Expenditures (million USD)	1.93	0.13	25.64	5.27	0.28	55.61	4.00	0.50	35.29
Age	15.22	14.00	9.46	16.36	14.00	11.81	15.20	14.00	11.35
Import Share (%)	3.84	0.00	13.62	3.20	0.00	11.14	3.50	0.00	12.55
Export Share (%)	1.14	0.00	9.28	0.48	0.00	4.30	0.63	0.00	6.80
Capital Share of Expenditures	0.17	0.05	0.24	0.12	0.02	0.19	0.16	0.07	0.21
Observations	23,890			2,642			3,053		

Notes: This table reports summary statistics about the size, age, capital intensity, and trade exposure of buyers, sellers in the sample, and sellers not in the sample for the year 2016, separated by seller's sector. Monetary values are in U.S. dollars for 2016.

for all studied industries.

Third, quantities increase as relationships age, both measured as standardized quantities or total quantity (Subfigure OA-6c and OA-6i; Subfigure OA-7c and OA-7i; Subfigure OA-8c and OA-8i). Quantities grow faster in textiles than in other industries. Moreover, looking at both standardized quantity and total quantity demanded, buyers tend to buy more of the same product over time and in total. In pharmaceutical products, product-specific demand levels off after the first year, while total demand continues to increase; in cement-products, product-specific demand levels off after the second year, while total demand continues to increase. In any case, quantity backloading appears relevant across the board.

Fourth, quantity discounts are observed, both within product and in average prices (Subfigure OA-6d and OA-6g; Subfigure OA-7d and OA-7g; Subfigure OA-8d and OA-8g). Thus, a model with price discrimination in quantities is important.

Fifth, price discounts tend to be offered to older relationships (Subfigure OA-6d and OA-6h; Subfigure OA-7d and OA-7h; Subfigure OA-8d and OA-8h). However, in contrast to the main figure, product-specific discounts are not observed on average in pharmaceuticals, whereas they are present in textiles and cement-products. In terms of average prices, relational discounts are observed across all industries. The contrast between quality-adjusted prices and average prices for pharmaceuticals indicates that product bundles are likely switching over time, allowing buyers to purchase cheaper products either not available or desired at the beginning of the relationship. In any case, a model with dynamic discounts could rationalize observed dynamics for average prices for all industries, as well as for quality-adjusted prices for textiles and cement.

Sixth, relationships that trade more intensively are more likely to survive across all industries (Subfigure OA-6f; Subfigure OA-7f; Subfigure OA-8f), though the heterogeneity across ages is smaller in pharmaceutical products than in textiles and cement.

Online Appendix Table OA-9 shows price dynamics by payment modality, relying on the transaction-level data. Joining all industries together (Column 1), we observe that for transactions conducted via trade-credit, quality-adjusted prices decrease as relationships age, accounting for plausible quantity discounts by controlling for a flexible spline in quantity. Instead, when the transaction's modality is pay-in-advance (Column 2), standardized prices increase as relationships age. The same pattern holds for textiles (Columns 3 and 4), cement-products (Columns 7 and 8), and even so for pharmaceuticals (Columns 5 and 6), where quality-adjusted pair-specific prices do not decrease over time.

Online Appendix Figure OA-9 shows the distribution of trade-credit terms offered by industry. Textiles offer on average 40 days of trade-credit, with 7, 30, 45, and 60 days as common terms. Pharmaceutical products offer on average 55 days, with 30, 45, and 60 as common terms. Cement-products offer 40 days, with 30, 45, and 60 days as common terms.

Finally, Online Appendix Table OA-10 presents coefficients of variation of sales and expenditures (month-to-month) for sellers and buyers. We can see that sellers have lower variability both in sales and expenditures than buyers, though variability is still present for sellers, as the standard deviation is 25% of the mean sales for pharmaceuticals, 29% for cement-products, and 42% for textiles. Production expenses are also volatile, with the standard deviation representing 30% of mean expenditures, though the difference across industries is much more muted than in sales.

Table OA-9: **Price Dynamics and Payment Method**

	All		Textiles		Pharmaceuticals		Cement-Products	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Payment Method	TC	O	TC	O	TC	O	TC	O
Total Years	-0.00786***	0.00431***	-0.00358***	0.00380***	-0.00641***	0.00770***	-0.0291***	-0.00218
	(0.00214)	(0.00108)	(0.00125)	(0.00110)	(0.00215)	(0.00232)	(0.0106)	(0.00174)
Observations	3,383,399	608,318	2,249,157	305,517	742,940	240,995	391,302	61,806
R-squared	0.954	0.982	0.988	0.977	0.981	0.975	0.758	0.973
Product-Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quantity Control	Spline	Spline	Spline	Spline	Spline	Spline	Spline	Spline

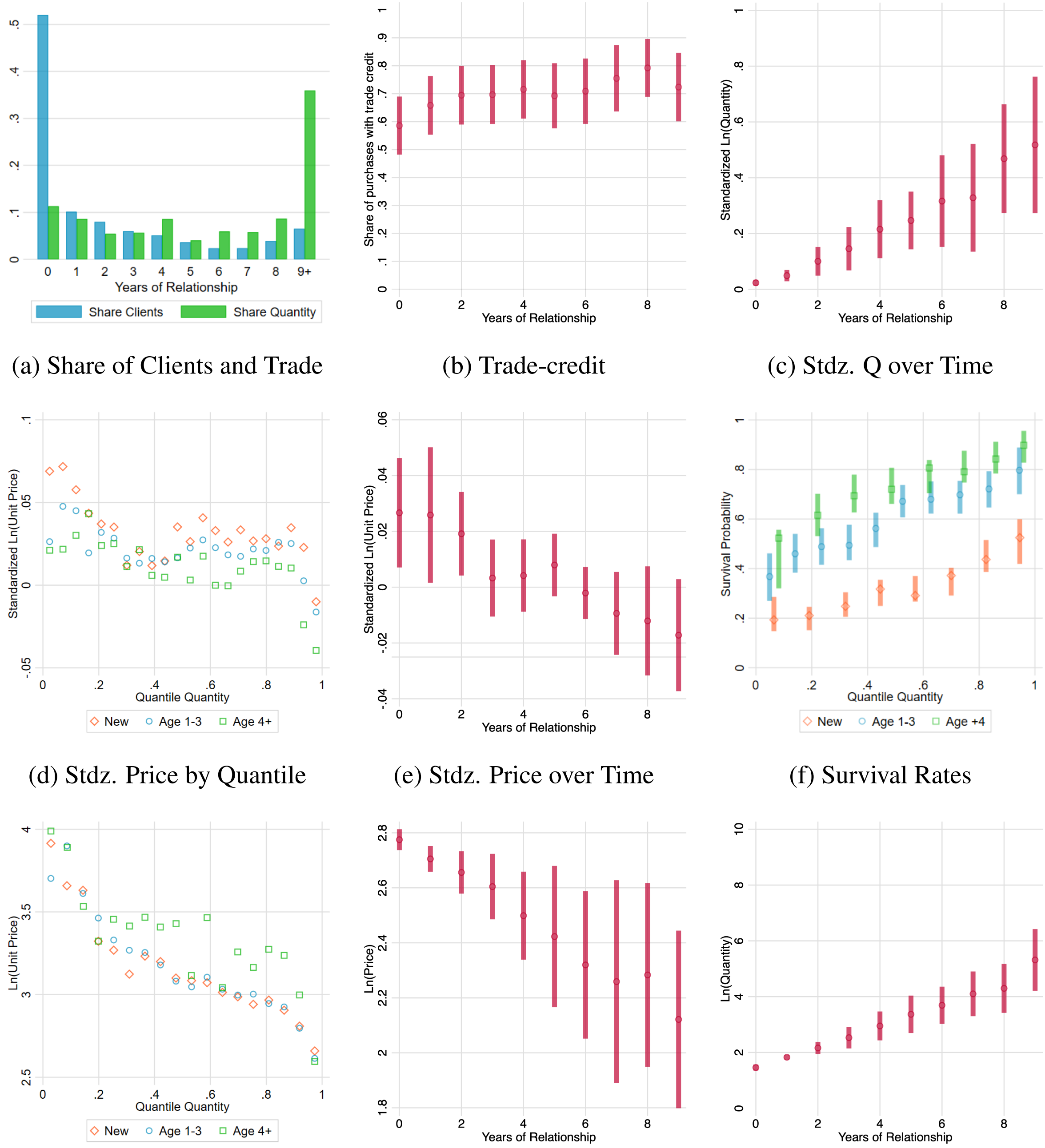
Notes: This table presents transactions-level regression of log unit prices on the age of relationship, controlling for a flexible spline of quantity and product-year fixed effects, by payment modality and sector. Columns (1) and (2) present results for all sectors, for trade-credit transactions and all others, respectively. Columns (3) and (4) report results for textiles, Columns (5) and (6) for pharmaceuticals, and Columns (7) and (8) for cement-products. Standard errors are clustered at the pair-year level. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table OA-10: **Coefficient of Variation of Sales and Expenditures**

	CV Sales Seller	CV Expenditures Seller	CV Sales Buyer	CV Expenditures Buyer
Textiles	0.42	0.28	0.65	0.65
Pharmaceuticals	0.25	0.34	0.77	0.55
Cement-Products	0.29	0.31	0.86	0.68

Notes: This table presents coefficients of variation (CV) in monthly sales and expenditures for sellers and buyers between 2016 and 2017.

Figure OA-6: **Motivating Facts: Textile-Products**



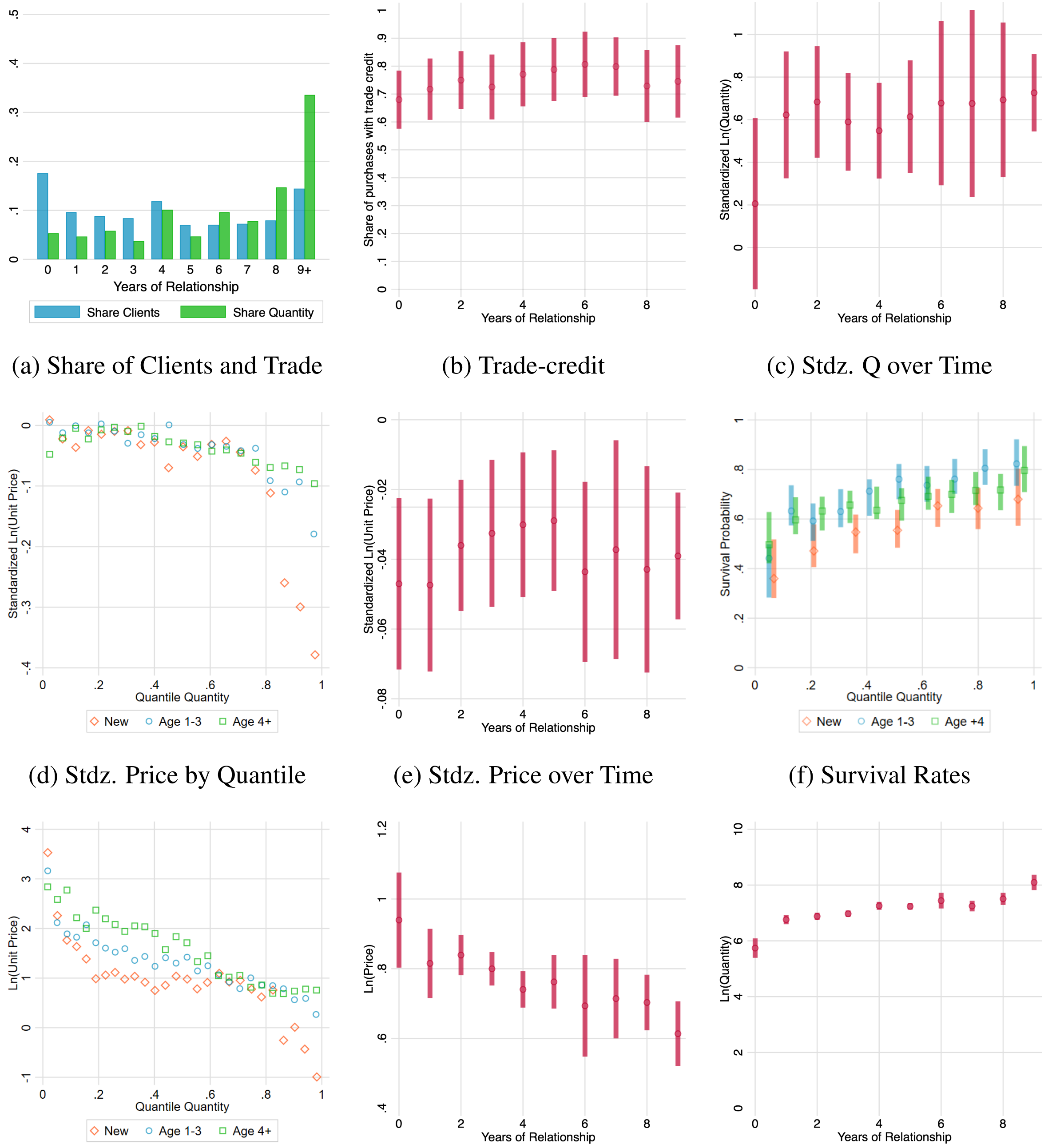
(a) Share of Clients and Trade (b) Trade-credit (c) Stdz. Q over Time

(d) Stdz. Price by Quantile (e) Stdz. Price over Time (f) Survival Rates

(g) ln(Price) by Quantile (h) ln(Price) over Time (i) ln(Q) over Time

Notes: This figure replicates Figure 1 for Textile-Products only. Subfigure (a) displays the distribution of the average of the share of clients and quantity sold by relationship age, calculated across all sellers in 2016. Subfigure (b) displays the average of the share of purchases channeled through trade-credit, along with a 90% confidence interval, calculated across all sellers. Subfigure (c) displays the evolution of standardized log quantities, with their corresponding 90% confidence intervals, calculated across all sellers. The standardized log quantity is obtained by taking the average quantity sold in a given year for each seller-product and subtracting the log average quantity for that year. The standard errors are calculated at the seller-year level. Subfigure d) shows the relationship between quantity purchased and standardized log unit price through a binscatter plot that displays the measure of unit price against the quantity sold, based on relationship age. The standardized log unit price is obtained by netting out the average log unit price for that year for each seller-product. The quantiles of quantity are calculated for each seller-relationship age combination. Subfigure e) presents a binscatter plot of standardized log unit prices against years of relationship, controlling for a flexible spline of standardized log quantities. The standard errors are calculated at the seller-year level. Subfigure f) displays a binscatter plot of the average survival rate of pairs at different ages and quantiles of quantity. Subfigure g) presents a binscatter of (log) average price on the quantile of quantity by relationship age, controlling for seller-year fixed effects. Subfigure h) presents a binscatter of (log) average price on years of relationship controlling for a flexible spline of quantity and seller-year fixed effects. Subfigure i) presents a binscatter of (log) total quantity on years of relationship controlling for seller-year fixed effects. The quantiles of quantities are calculated for each seller-age combination, and the error bars represent a 90% level of variation across all sellers.

Figure OA-7: **Motivating Facts: Pharmaceutical-Products**



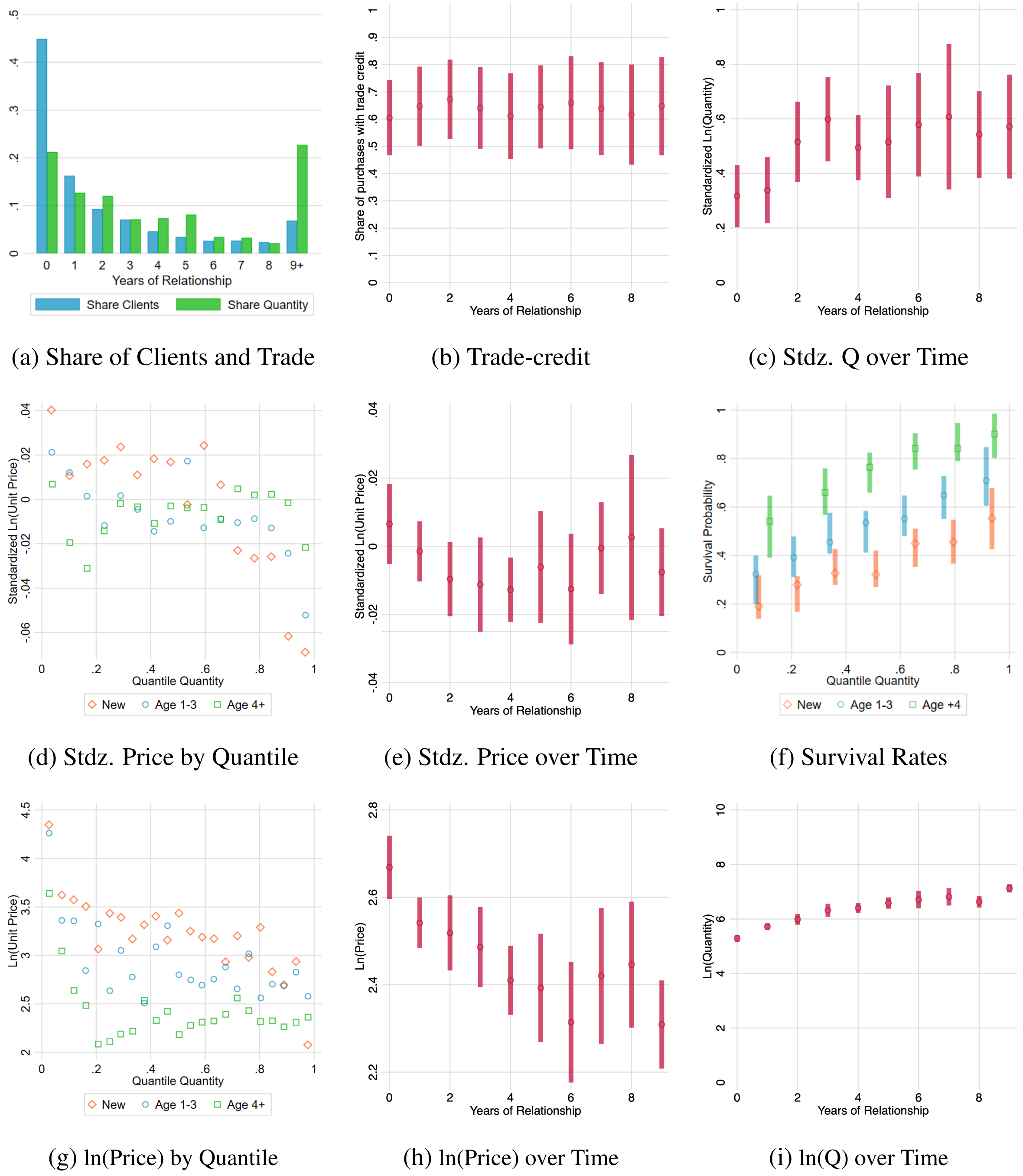
(a) Share of Clients and Trade (b) Trade-credit (c) Stdz. Q over Time

(d) Stdz. Price by Quantile (e) Stdz. Price over Time (f) Survival Rates

(g) ln(Price) by Quantile (h) ln(Price) over Time (i) ln(Q) over Time

Notes: This figure replicates Figure 1 for Pharmaceutical-Products only. Subfigure (a) displays the distribution of the average of the share of clients and quantity sold by relationship age, calculated across all sellers in 2016. Subfigure (b) displays the average of the share of purchases channeled through trade-credit, along with a 90% confidence interval, calculated across all sellers. Subfigure (c) displays the evolution of standardized log quantities, with their corresponding 90% confidence intervals, calculated across all sellers. The standardized log quantity is obtained by taking the average quantity sold in a given year for each seller-product and subtracting the log average quantity for that year. The standard errors are calculated at the seller-year level. Subfigure d) shows the relationship between quantity purchased and standardized log unit price through a binscatter plot that displays the measure of unit price against the quantity sold, based on relationship age. The standardized log unit price is obtained by netting out the average log unit price for that year for each seller-product. The quantiles of quantity are calculated for each seller-relationship age combination. Subfigure e) presents a binscatter plot of standardized log unit prices against years of relationship, controlling for a flexible spline of standardized log quantities. The standard errors are calculated at the seller-year level. Subfigure f) displays a binscatter plot of the average survival rate of pairs at different ages and quantiles of quantity. Subfigure g) presents a binscatter of (log) average price on the quantile of quantity by relationship age, controlling for seller-year fixed effects. Subfigure h) presents a binscatter of (log) average price on years of relationship controlling for a flexible spline of quantity and seller-year fixed effects. Subfigure i) presents a binscatter of (log) total quantity on years of relationship controlling for seller-year fixed effects. The quantiles of quantities are calculated for each seller-age combination, and the error bars represent a 90% level of variation across all sellers.

Figure OA-8: **Motivating Facts: Cement-Products**



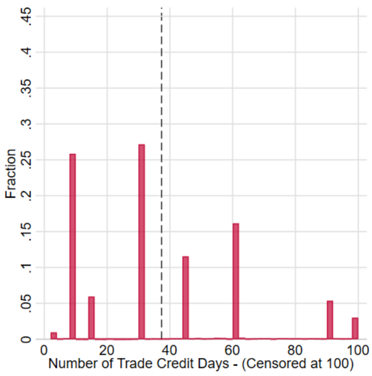
(a) Share of Clients and Trade (b) Trade-credit (c) Stdz. Q over Time

(d) Stdz. Price by Quantile (e) Stdz. Price over Time (f) Survival Rates

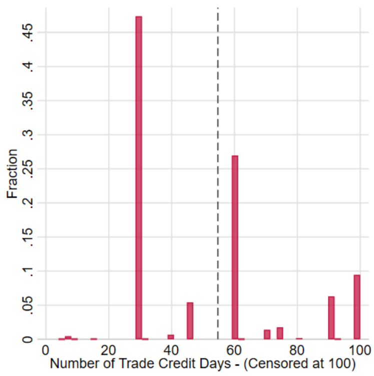
(g) ln(Price) by Quantile (h) ln(Price) over Time (i) ln(Q) over Time

Notes: This figure replicates Figure 1 for Cement-Products only. Subfigure (a) displays the distribution of the average of the share of clients and quantity sold by relationship age, calculated across all sellers in 2016. Subfigure (b) displays the average of the share of purchases channeled through trade-credit, along with a 90% confidence interval, calculated across all sellers. Subfigure (c) displays the evolution of standardized log quantities, with their corresponding 90% confidence intervals, calculated across all sellers. The standardized log quantity is obtained by taking the average quantity sold in a given year for each seller-product and subtracting the log average quantity for that year. The standard errors are calculated at the seller-year level. Subfigure d) shows the relationship between quantity purchased and standardized log unit price through a binscatter plot that displays the measure of unit price against the quantity sold, based on relationship age. The standardized log unit price is obtained by netting out the average log unit price for that year for each seller-product. The quantiles of quantity are calculated for each seller-relationship age combination. Subfigure e) presents a binscatter plot of standardized log unit prices against years of relationship, controlling for a flexible spline of standardized log quantities. The standard errors are calculated at the seller-year level. Subfigure f) displays a binscatter plot of the average survival rate of pairs at different ages and quantiles of quantity. Subfigure g) presents a binscatter of (log) average price on the quantile of quantity by relationship age, controlling for seller-year fixed effects. Subfigure h) presents a binscatter of (log) average price on years of relationship controlling for a flexible spline of quantity and seller-year fixed effects. Subfigure i) presents a binscatter of (log) total quantity on years of relationship controlling for seller-year fixed effects. The quantiles of quantities are calculated for each seller-age combination, and the error bars represent a 90% level of variation across all sellers.

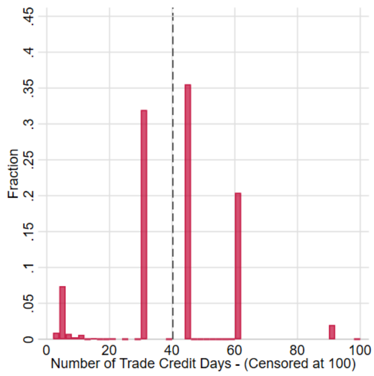
Figure OA-9: **Trade-credit Terms by Sector**



(a) Textiles



(b) Pharmaceutical



(c) Cement-Products

Notes: This figure plots the distribution of trade-credit days offered by the seller's sector.

OA-4 Model Propierties and a Solved Example

OA-4.1 Existence and Non-Stationarity

To prove existence, I build on two key results from the literature. First, I utilize the result of non-linear pricing from Jullien (2000) to demonstrate the existence of a stationary optimal contract in the presence of heterogeneous participation constraints. This is achieved by showing the equivalence between the stationary contract with limited enforcement and a non-linear pricing problem with heterogeneous outside options. Subsequently, similar to the argument in Martimort et al. (2017), I present a simple non-stationary deviation that outperforms the stationary optimal contract.

It is important to note that I will show existence results under the assumption of no exit, i.e., $X(\theta)=0$ for all θ. To prove existence with exit, one must replace the discount factor δ with $\tilde{\delta}\equiv\min\{\delta(\theta)\}$, where $\delta(\theta)=\delta(1-X(\theta))$ accounts for heterogeneous breakups. This adjustment only affects one of the assumptions discussed below and sets an upper bound on the worst-case exit rate.

OA-4.1.1 *Existence of Stationary Contract*

The model in Jullien (2000) solves the following problem:

$$\max_{\{t(\theta),q(\theta)\}}\int_{\underline{\theta}}^{\overline{\theta}}[t(\theta)-cq(\theta)]f(\theta)d\theta \ \text{ s.t.} \tag{IR Problem}$$

$$v(\theta,q(\theta))-t(\theta)\geq v(\theta,q(\hat{\theta}))-t(\hat{\theta}) \ \ \forall\theta,\hat{\theta} \tag{IC}$$

$$v(\theta,q(\theta))-t(\theta)\geq\overline{u}(\theta) \ \ \forall\theta. \tag{IR}$$

Under a modified first-order approach, the seller's first-order condition is given by:

$$v_q(\theta,q(\theta))-c=\frac{\gamma(\theta)-F(\theta)}{f(\theta)}v_{\theta q}(\theta,q(\theta)), \tag{39}$$

for each type θ, and the complementary slackless condition on the IR constraints:

$$\int_{\underline{\theta}}^{\overline{\theta}}[u(\theta)-\overline{u}(\theta)]d\gamma(\theta)=0. \tag{40}$$

Jullien (2000) shows that under three assumptions there exists a unique optimal solution in which all consumers participate. This solution is characterized by the first-order conditions 39 and complementary slackless condition 40 with $q(\theta)$ increasing.

The first assumption is potential separation (PS), which requires that the optimal solution is non-decreasing in θ, and satisfied under weak assumptions on the distribution of θ and the curvature of the surplus relative to the return of the buyer. In particular, it requires that

$$\frac{d}{d\theta}\left(\frac{S_q(\theta,q)}{v_{\theta q}(\theta,q)}\right)\geq 0$$

$$\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)\geq 0\geq\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right).$$

The second and *key* assumption is homogeneity (H), requiring that there exists a quantity profile $\{\overline{q}(\theta)\}$ such that the allocation with full participation $\{\overline{u}(\theta),\overline{q}(\theta)\}$ is implementable in that $\overline{u}'(\theta)=v_\theta(\theta,\overline{q}(\theta))$ and $\overline{q}(\theta)$ is weakly increasing. This assumption implies that the reservation return can be implemented as a contract without excluding any type, ensuring that incentive compatibility is not an issue when the individual rationality constraint is binding.

Lastly, the assumption of full participation (FP) posits all types participate, and is satisfied when (H) holds and the surplus generated in the reservation return framework is greater than the private return to the buyer, i.e. $s(\theta,\overline{q}(\theta)) \geq \overline{u}(\theta)$.

I show that my setting can be rewritten in terms of Jullien (2000), implying that an optimal separating stationary contract exists. The seller chooses the optimal stationary contract $\{t(\theta),q(\theta)\}$ that satisfy incentive-compatibility and the limited enforcement constraint. Formally, the seller solves the problem:

$$\max_{\{t(\theta),q(\theta)\}} \frac{1}{1-\delta}\int_{\underline{\theta}}^{\overline{\theta}} [t(\theta)-cq(\theta)]f(\theta)d\theta \text{ s.t.} \tag{LE Problem}$$

$$v(\theta,q(\theta))-t(\theta) \geq v(\theta,q(\hat{\theta}))-t(\hat{\theta}) \ \ \forall\theta,\hat{\theta} \tag{IC}$$

$$\frac{\delta}{1-\delta}\Big(v(\theta,q(\theta))-t(\theta)\Big) \geq t(\theta) = v(\theta,q(\theta))-u(\theta), \ \ \forall\theta, \tag{LC}$$

where $u(\theta)$ is the return obtained by type θ. The limited enforcement constraint can be easily written as the IR constraint in Jullien (2000):

$$u(\theta) \geq (1-\delta)v(\theta,q(\theta)) \equiv \overline{u}(\theta) \ \ \forall\theta. \tag{LE'}$$

In my model, with $v(\theta,q) = \theta v(q)$, the first condition of assumption PS is always satisfied as

$$\frac{d}{d\theta}\left(\frac{S_q(\theta,q)}{v_{\theta q}(\theta,q)}\right) = \frac{d}{d\theta}\left(\theta - \frac{c}{v'(q)}\right) \geq 0 \Longleftrightarrow 1 \geq 0 \tag{A1}$$

As stated earlier, the second condition of assumption PS is satisfied for a wide-range of distributions for θ. Therefore, assumption PS is satisfied for any of those distributions.

Then, consider Assumption H. It requires that an allocation $\{\overline{q}(\theta)\}$ exists such that $\overline{u}'(\theta) = v_\theta(\theta,\overline{q}(\theta))$ and $\overline{q}(\theta)$ is weakly increasing. Notice that under LE', we can define $\overline{q}(\theta)$ as $\overline{u}'(\theta) = (1-\delta)[\theta v'(q(\theta))q'(\theta)+v(q(\theta)] = v(\overline{q}(\theta))$. Define $G(\overline{q},\theta) = v(\overline{q})-(1-\delta)[\theta v'(q(\theta))q'(\theta)+v(q(\theta))] = 0$. By the implicit function theorem, $\overline{q}(\theta)$ is weakly increasing if

$$\begin{aligned}
\overline{q}'(\theta) &= -\frac{dG/d\theta}{dG/d\overline{q}} \\
&= \frac{(1-\delta)[v'(q(\theta))q'(\theta)+\theta v''(q(\theta))(q'(\theta))^2+\theta v'(q(\theta))q''(\theta)+v'(q(\theta))]}{v'(\overline{q})} \geq 0 \\
&\Longleftrightarrow v'(q(\theta))[1+q'(\theta)+\theta q''(\theta))]+\theta v''(q(\theta))(q'(\theta))^2 \geq 0 \\
&\Longleftrightarrow \frac{q'(\theta)+\theta q''(\theta)+1}{\theta(q'(\theta))^2} \geq A(q) \\
&\Longleftrightarrow \left(\frac{T''(q)}{T'(q)}+A(q)\right)\Big(1+\theta(q)\theta'(q)r(q)+\theta'(q)\Big) \geq A(q) \\
&\Longleftrightarrow \frac{T''(q)}{T'(q)}\frac{M(q)}{M(q)-1} \geq A(q),
\end{aligned}$$

where $M(q) \equiv 1+\theta(q)\theta'(q)r(q)+\theta'(q)$ and $r(q) = g^{-1}(q)$ for $g(\theta) \equiv q''(\theta)$. As we expect

$T''(q) < 0$ and $T'(q) > 0$, it is necessary that $M(q)/(M(q)-1) < 0$. Such condition will be satisfied if $M(q) < 1$ and $M(q) > 0$, which imply that

$$\begin{gathered} r(q)\theta(q) < -1 \\ \text{and} \\ \theta'(q) < \frac{1}{\theta(q)|r(q)|-1}. \end{gathered} \tag{A2}$$

The first condition sets restrictions on the rate of change of quantities, which requires $q''(\theta)$ to be negative, restricting how convex $u(\theta)$ can be. The second condition requires that quantities increase at a minimum rate. Moreover, the condition sets bounds on the price discounts offered relative to the buyers' return curvature at a given quantity.

Lastly, full participation requires H to hold as well as $s(\theta,\overline{q}(\theta)) \geq (1-\delta)\theta v(\overline{q}(\theta))$. The condition becomes:

$$\delta \geq \frac{c\overline{q}(\theta)}{\theta v(\overline{q}(\theta))}, \tag{A3}$$

which requires that agents value the future high enough, such that discount factor be greater than the ratio of average cost to average return.

Let $\{t^{st}(\theta), q^{st}(\theta)\}$ be the solution to the to the problem characterized by equations 39 and 40. Assuming that the primitives $v(\cdot)$, $F(\theta)$, and δ are such that conditions A1, A2, and A3 hold for $\{t^{st}(\theta), q^{st}(\theta)\}$, then $\{t^{st}(\theta), q^{st}(\theta)\}$ is uniquely optimal.

OA-4.1.2 *Solution to Stationary* $\Gamma^{st}(\theta)$

The seller's first-order condition defines the following differential equation in the stationary equilibrium

$$\theta u'(q^{st}(\theta)) - c = \frac{\Gamma^{st}(\theta) - F(\theta) + (1-\delta)\theta\gamma^{st}(\theta)}{f(\theta)} u'(q^{st}(\theta)). \tag{41}$$

The solution $\Gamma^{st}(\theta)$ to the equation above is given by:

$$\Gamma^{st}(\theta) = \frac{\int_{\underline{\theta}}^{\theta} x^{\delta/(1-\delta)}[xf(x) - c(u'(q^{st}(x))^{-1}f(x) + F(x)]dx + K}{\theta^{1/(1-\delta)}(1-\delta)}, \tag{42}$$

which by integration by parts reduces to:

$$\Gamma^{st}(\theta) = \frac{F(\theta)}{1-\delta} - \frac{\delta\int^{\theta} x^{\delta/(1-\delta)}F(x)dx}{(1-\delta)\theta^{1/(1-\delta)}} - \frac{cE[x^{\delta/(1-\delta)}u'({}^{st}q(x))^{-1}|x \leq \theta]}{(1-\delta)\theta^{1/(1-\delta)}} + \frac{K}{(1-\delta)\theta^{1/(1-\delta)}} \tag{43}$$

The constant is obtained by using the boundary condition $\Gamma^{st}(\overline{\theta}) = 1$. Therefore,

$$K = cE[x^{\delta/(1-\delta)}u'(q^{st}(x))^{-1})] - \delta\overline{\theta}^{1/(1-\delta)} + \delta \int x^{\delta/(1-\delta)}F(x)dx. \tag{44}$$

OA-4.1.3 *Optimality of Non-Stationary Contracts*

Having established the existence of an optimal stationary contract, I show the optimality of non-stationary contracts.

Proposition 3. *If a non-stationary optimal contract exists, then it dominates the optimal stationary contract.*

Proof of Proposition 2. Consider the following deviation from the stationary contract, in which at tenure 0, the return obtained by the buyer is given by:

$$u_0(\theta) = u^{st}(\theta) - \varepsilon,$$

for some $\varepsilon > 0$ sufficiently small, where $u^{st}(\theta) = \theta v(q^{st}(\theta)) - t^{st}(\theta)$ and $t_0(\theta) = t^{st}(\theta)$. Define $q_0(\theta)$ to satisfy this deviation. Under this deviation, the enforcement constraint at $\tau = 0$ is:

$$t^{st}(\theta) \leq \frac{\delta}{1-\delta}\left[\theta v(q^{st}(\theta)) - t^{st}(\theta)\right],$$

which is identical to the one in the stationary contract, which we know $\{t^{st}(\theta), q^{st}(\theta)\}$ satisfy. Moreover, the incentive compatibility constraint is still satisfied as $\hat{\theta}$ maximizes

$$u_0(\theta,\hat{\theta}) + \frac{\delta}{1-\delta}u^{st}(\theta,\hat{\theta}) = \frac{\delta}{1-\delta}u^{st}(\theta,\hat{\theta}) - \varepsilon,$$

where $u_\tau(\theta,\hat{\theta}) \equiv \theta v(q_\tau(\hat{\theta})) - t_\tau(\hat{\theta})$.

Under this alternative scheme, the seller obtains an additional payoff ε per buyer while still satisfying both the incentive compatibility and limited enforcement constraints. Therefore, if it exists, the optimal non-stationary contract dominates the optimal stationary one. □

OA-4.2 Model Dynamics

Quantity Discounts

Define $T_\tau(q_\tau(\theta)) \equiv t_\tau(\theta_\tau(q))$, $\Lambda_\tau(\theta) \equiv \Gamma_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)$, and $\lambda_\tau(\theta) \equiv d\Lambda_\tau/d\theta$. The price schedule is said to feature quantity discounts if $T''_\tau(q) < 0$.

Proposition 4. *Assume strict monotonicity of quantity $q'_\tau(\theta) > 0$ and that $\lambda_\tau(\theta) < f_\tau(\theta)$. If the densities $f_\tau(\theta)$ satisfy log-concavity and $d(F_\tau(\theta)/f_\tau(\theta))/d\theta \geq F_\tau(\theta)/[(\theta-1)f_\tau(\theta)]$, then the tariff schedule exhibits quantity discounts, $T''_\tau(q) \leq 0$ for each $q = q_\tau(\theta)$, $\theta \in (\underline{\theta}, \overline{\theta})$ and τ.*

Proof of Proposition 3. Recall the quantity function $q_\tau(\theta)$ and its inverse function $\theta_\tau(q)$. Further differentiating the derivative of the incentive-compatible tariff schedule $T'_\tau(q_\tau(\theta)) = \theta v'(q_\tau(\theta))$ gives:

$$T_\tau''(q) = \theta_\tau'(q)v'(q) + \theta_\tau(q)v''(q) = \theta(q)v'(q)\left[\frac{\theta_\tau'(q)}{\theta_\tau(q)} + \frac{v''(q)}{v'(q)}\right] \tag{45}$$

$$= T'(q)\left[\frac{1}{\theta_\tau(q)q_\tau'(\theta)} - A(q)\right], \tag{46}$$

for $A(q) = -v''(q)/v'(q)$ and $\theta_\tau'(q) = 1/q_\tau'(\theta)$.

By implicit differentiation on the seller's first-order condition, we obtain an expression for $q_\tau'(\theta)$:

$$q_\tau'(\theta) = -\frac{\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]v'(q_\tau(\theta))}{\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]v''(q_\tau(\theta))}$$

$$= \frac{1}{A(q_\tau(\theta))}\frac{\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}{\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}$$

From SFOC, the denominator of the equation above is positive as $v'(q_\tau(\theta)) > 0$ and $c > 0$. By assumption, strict monotonicity holds ($q_\tau'(\theta) > 0$), which implies that the numerator is also positive. Substituting into (45) and using the fact that $T_\tau'(q) > 0$ and $A(q_\tau) > 0$, quantity discounts $T_\tau''(q) \leq 0$ hold if and only if

$$\frac{\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}{\theta\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]} \leq 1 \tag{47}$$

Inequality 47 holds if

$$\theta - \frac{\Lambda_\tau(\theta) - F_\tau(\theta)}{f_\tau(\theta)} \leq \theta - \theta\frac{(\lambda_\tau(\theta) - f_\tau(\theta))f_\tau(\theta) - (\Lambda_\tau(\theta) - F_\tau(\theta))f_\tau'(\theta)}{f_\tau(\theta)^2}.$$

Rearranging, one obtains

$$[\Lambda_\tau(\theta) - F_\tau(\theta)][f_\tau(\theta) + f_\tau'(\theta)\theta] \geq \theta f(\theta)[\lambda_\tau(\theta) - f_\tau(\theta)]. \tag{48}$$

From the positive denominator above, one can obtain that $\theta f_\tau(\theta) \geq \Lambda_\tau(\theta) - F_\tau(\theta)$. Moreover, note that the log-concavity of the density $F_\tau(\theta)$ is sufficient to satisfy the standard assumption of the monotone hazard condition. So concentrating on log-concave densities, the following inequality holds: $f_\tau(\theta) \geq f_\tau'(\theta)\theta$. Therefore, if $\Lambda_\tau(\theta) > F_\tau(\theta)$, then a sufficient condition for quantity discounts is $\lambda_\tau(\theta) < f_\tau(\theta)$.

Instead if $\Lambda_\tau(\theta) < F_\tau(\theta)$, one can write 48 as

$$(\theta - 1)f_\tau(\theta) + f_\tau(\theta) \geq [F_\tau(\theta) - \Lambda_\tau(\theta)]\left(1 + \frac{f_\tau'(\theta)\theta}{f_\tau(\theta)}\right) + \lambda_\tau(\theta). \tag{49}$$

If $f_\tau'(\theta) < 0$, then a sufficient condition is $(\theta - 1)f_\tau(\theta) \geq F_\tau(\theta)$. If $f_\tau'(\theta) > 0$, then a sufficient condition is that $(\theta - 1)f(\theta) \geq F_\tau(\theta)(1 + \theta f_\tau'(\theta)/f_\tau(\theta))$. Both conditions can be expressed as:

$$\frac{d}{d\theta}\left(\frac{F_\tau(\theta)}{f_\tau(\theta)}\right) = \frac{f_\tau(\theta)^2 - F_\tau(\theta)f'_\tau(\theta)}{f_\tau(\theta)^2} \geq \frac{F_\tau(\theta)}{(\theta-1)f_\tau(\theta)}. \tag{50}$$

$\square$

Intuitively, the condition states that for a general class of distributions, as long as the incentive-compatibility marginal effects dominate those of the limited enforcement, the seller finds it optimal to offer quantity discounts at any relationship age. This condition is likely to be satisfied if the limited enforcement constraint is slack for some buyers even at their first interaction. Moreover, it also requires the enforcement constraint to be slack for all buyers in the long run. This last requirement aligns with the model of Martimort et al. (2017), where buyers reach a *mature* phase in which the constraints no longer bind. This is also consistent with Proposition 4 below, which finds that trade reaches a mature phase.

In terms of generality, the usual monopolist screening problem requires (or uses) log-concavity of $f(\theta)$.[48] I am strengthening the requirement that the evolution of the distribution also satisfies log-concavity, implicitly placing bounds on the distribution of exit rates over types.

The second condition strengthens the requirements on the dynamic distribution of types to ensure that the seller desires to price discriminate across types.

An alternative way to consider this property is to use (t-RULE) to obtain that the tariff schedule is concave if and only if $q'_\tau(\theta) > \frac{v'(q_\tau(\theta))}{-v''(q_\tau(\theta))\theta}$. As long as quantities increase by types fast enough, the seller will offer quantity discounts. The rate at which the quantities have to increase is determined by the level of the type and the curvature of the return function.

Evolution of Quantities

Next, I discuss how quantities evolve in Proposition 4.

Proposition 5. *For each θ, quantity increases monotonically in τ (i.e., $q_\tau(\theta) \leq q_{\tau+1}(\theta)$) if and only if the limited enforcement constraint is relaxed over time ($\gamma_\tau(\theta) \geq \gamma_{\tau+1}(\theta)$). Moreover, there is a time τ^* such that $\forall \tau \geq \tau^*$, $\gamma_{\tau^*}(\theta) = 0$ for all $\theta > \underline{\theta}$ and $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ for all $\tau < \tau^*$ and all θ.*

Proof of Proposition 4. Notice that by the seller's first-order condition and $v'(\cdot) > 0$, $q_\tau(\theta) \leq q_{\tau+1}(\theta)$ holds if and only if

$$\begin{aligned} V_\tau(\theta) &\equiv \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)} \\ &\geq \frac{f_\tau(\theta)}{f_{\tau+1}(\theta)} \frac{\Gamma_\tau(\theta) - F_{\tau+1}(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_{\tau+1}(\theta)}{f_\tau(\theta)} + \frac{\Gamma_{\tau+1}(\theta)-1}{f_{\tau+1}(\theta)} \equiv V_{\tau+1}(\theta)v, \end{aligned}$$

which can be written as

$$V_\tau(\theta) \geq \frac{f_\tau(\theta)}{f_{\tau+1}(\theta)}V_\tau(\theta) + \frac{\Gamma_{\tau+1}(\theta)-1}{f_{\tau+1}(\theta)} + \frac{\theta[\gamma_{\tau+1}(\theta)-\gamma_\tau(\theta)]}{f_{\tau+1}(\theta)} - \frac{F_{\tau+1}(\theta)-F_\tau(\theta)}{f_{\tau+1}(\theta)}.$$

[48] Log-concavity of a density function $g(x)$ is equivalent to $g'(x)/g(x)$ being monotone decreasing. Families of density functions satisfying log-concavity include: uniform, normal, extreme value, exponential, amongst others.

With no selection pattern, i.e. $f_\tau(\theta) = f_{\tau+1}(\theta)$, the condition reduces to

$$\frac{1-\Gamma_{\tau+1}(\theta)}{f_\tau(\theta)} \geq \frac{\theta[\gamma_{\tau+1}(\theta)-\gamma_\tau(\theta)]}{f_\tau(\theta)}.$$

As $\gamma_\tau(\theta) > 0$ by assumption and the left-hand side is (weakly) positive due to $\Gamma_{\tau+1}(\theta) \leq 1$, a sufficient condition is that $\gamma_{\tau+1}(\theta) < \gamma_\tau(\theta)$. To obtain necessity, consider the Lagrangian keeping future return U^+ constant. The seller chooses $q(\theta)$ maximizing the following program:

$$L(\theta, U, q, \lambda, \gamma) = (\theta v(q(\theta)) - cq(\theta) - U)f(\theta) + \lambda v(q(\theta)) + \gamma(U + \delta U^+ - \theta v(q(\theta))), \quad (51)$$

where λ is the co-state variable for the incentive-compabilitity constraint and γ is the multiplier for the limited enforcement constraint. Noting that the necessary conditions are also sufficient (Seierstad and Sydsaeter, 1986) (pg. 276), the relevant optimality conditions are:

$$f(\theta)[\theta v'(q(\theta)) - c] + \lambda(\theta)v'(q(\theta)) = \gamma(\theta)\theta v'(q(\theta))$$

and

$$\dot{\lambda}(\theta) = f(\theta) - \gamma(\theta)$$

which imply

$$\gamma(\theta) = f(\theta) - \frac{cf(\theta)}{\theta v'(q(\theta))} + \frac{F(\theta)-\Gamma(\theta)}{\theta}.$$

Therefore, a higher level of quantity $q(\theta)$ is implied by a lower $\gamma(\theta)$.

Next, to obtain that $\gamma_\tau(\theta) = 0$ for some finite $\tau > \tau^*$ for all $\theta > \underline{\theta}$. Suppose otherwise, such that $\gamma_\tau(\tilde{\theta}) > 0$ for some $\tilde{\theta}$ and all τ. Then, $\Gamma_\tau(\theta) < 1$ for all $\theta \leq \tilde{\theta}$. Therefore, $1 - \Gamma_\tau(\theta) > 0$ for all $\theta \leq \tilde{\theta}$. Thus, as $\tau \to \infty$, $\sum_{s=0}^{\tau}(1-\Gamma_s(\theta)) \to \infty$ for all $\theta \leq \tilde{\theta}$. Thus, as long as $q_\tau(\theta) < \infty$ for all θ, τ, it must be the case that some finite τ^* exists such that $\gamma_\tau(\theta) = 0$ for all $\tau > \tau^*$ and for all θ. It is possible however for enforcement constraints to bind for $\underline{\theta}$, as in that case $\Gamma_\tau(\underline{\theta}) = 1$ and quantities would be finite.

Finally, to obtain $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ for all $\tau < \tau^*$ and all θ. Notice that $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ if and only if

$$\theta\gamma_\tau(\theta) + \sum_{s=\tau+1}^{\tau^*-1}(1-\Gamma_s(\theta)) \geq 0,$$

which always holds. It holds with strict inequality whenever the enforcement constraint binds at period τ, or when it binds in some period between τ and τ^* for some θ between $\underline{\theta}$ and θ.

□

In the model, quantities go hand-in-hand with enforcement constraints. Although the exact path depends on further assumptions on the return function and the distribution of types, the model predicts that quantities will reach a mature phase in which constraints no longer bind, except perhaps for the lowest type. At this mature phase, quantities will be at their highest level in the relationship.

Discounts over time

The model also offers conditions under which discounts over time are observed.

Proposition 6. *If $M_{\tau+1}(\theta) \equiv q_{\tau+1}(\theta) - q_\tau(\theta) \geq 0$ for all θ and with strict inequality for $\underline{\theta}$, then $p_{\tau+1}(q) \equiv T_{\tau+1}(q)/q < T_\tau(q)/q \equiv p_\tau(q)$.*

Proof of Proposition 5. Use the marginal price function $T'_\tau(q) = \theta_\tau(q)v'(q)$. Average unit prices $p_\tau(q)$ for $q > 0$ are given by:

$$p_\tau(q) = \frac{T_\tau(q)}{q} = \frac{\int_0^q \theta_\tau(x)v'(x)dx}{q},$$

where I have used the normalization $T_\tau(0) = 0$ and the inverse function $\theta_\tau(q)$. Average prices decrease over time if and only if

$$\int_0^q \theta_\tau(x)v'(x)dx > \int_0^q \theta_{\tau+1}(x)v'(x)dx$$
$$\iff$$
$$\int_0^q [\theta_\tau(x) - \theta_{\tau+1}]v'(x)dx > 0.$$

By assumption, $q_\tau(\theta) \geq q_{\tau+1}(\theta)$ (and strictly so for $\underline{\theta}$). Thus, $\theta_\tau(q) > \theta_{\tau+1}(q)$ for all q and the inequality holds.

$\square$

As long as quantities (weakly) increase from τ to $\tau+1$, unit prices at any given q decrease. The intuition behind this result is that marginal prices match marginal returns. A right-ward shift in quantities for (some) buyers further lowers marginal returns, requiring a decrease in marginal prices as well. As such, average prices will be lower at each q as well.

To further understand the dynamics in the model, I present a solved two-type example in Online Appendix Section OA-4.4. The example illustrates the backloading of prices and quantities together with quantity discounts as a way to maximize lifetime profits for the seller while preventing opportunistic behavior from the buyer.

OA-4.3 Equilibrium Contracts under Relaxation of the Constraints

OA-4.3.1 *Perfect Enforcement and Complete Information*

Under complete information and full enforcement, the seller acts as a monopolist practicing first-degree price discrimination with a stationary contract $(t^{1d}(\theta), q^{1d}(\theta))$, defined as

$$\theta v'(q^{1d}(\theta)) = c \quad \text{and} \quad t^{1d}(\theta) = \theta v(q^{1d}(\theta)). \tag{1D-Q \& 1D-T}$$

The seller offers first-best quantities but extracts all the rents from the buyer (subject to an interim individual rationality constraint, $u_\tau(\theta) \geq 0$). This allocation is infinitely repeated over time. In this model, quantities and prices are constant over time, hence there are no dynamics. Moreover, while quantities increase by type, prices may be constant under some parametrizations of $v(\cdot)$.

OA-4.3.2 *Perfect Enforcement and Incomplete Information*

This setting is similar to the canonical repeated adverse selection problem (Baron and Besanko, 1984; Sugaya and Wolitzky, 2021). As the seller has commitment, there is no loss of generality in restricting the study to an infinite sequence menu $\{t_\tau(\theta), q_\tau(\theta)\}_{\underline{\theta},\overline{\theta}}$ that induces the agent to report their true type. The problem of the seller is maximizing profits subject to IC-B and interim individual rationality constraints ($u_\tau(\theta) \geq 0$).

The theoretical insights from Baron and Besanko (1984) apply in this setup.[49] The optimal dynamic contract with full enforcement is equal to repeated Baron-Myerson static contracts with quantities determined by:

$$\theta v'(q_\tau^{pe}) = c_\tau - \frac{1-F_\tau(\theta)}{f_\tau(\theta)} v(q_\tau^{pe}(\theta)), \tag{PE-Q}$$

and tariffs such that

$$t_\tau^{pe}(\theta) = \theta v(q_\tau^{pe}(\theta)) - \int_{\underline{\theta}}^{\theta} v(q_\tau^{pe}(x))dx. \tag{PE-T}$$

To preserve incentive compatibility of the buyer, the seller offers higher quantities to higher types within a given period. Moreover, the price schedule is shown to feature quantity discounts under common classes of assumptions on the curvature of demand and the distribution of types (Maskin and Riley, 1984) .

Under positive selection (i.e., $X'(\theta) < 0, \forall\theta$), average and type-specific quantities *decrease* over time. Similarly, average and type-specific unit prices *increase*.[50] Instead, without selection patterns (i.e., $X'(\theta) = 0, \forall\theta$), the optimal full enforcement contract with asymmetric information is stationary.

Therefore, while asymmetric information is able to rationalize the observed quantities discounts, on its own, it is not able to rationalize the dynamics of quantities and prices under observed selection patterns.

OA-4.3.3 *Limited Enforcement and Complete Information*

Next, consider a model without adverse selection, where the buyer can default on trade-credit at any time. In this context, the seller selects trade profiles $\{t_\tau(\theta), q_\tau(\theta)\}_{\underline{\theta},\overline{\theta}}$ that maximize lifetime profits, subject only to the limited enforcement constraint (equation LE-B). This model is reminiscent of the models in Thomas and Worrall (1994), Ray (2002), and Albuquerque and Hopenhayn (2004), which feature quantity and price backloading, as those described in the reduced form section.

In particular, the optimal contract quantities are determined by the following equation:

$$\theta v'(q_\tau^{le}) = \frac{c_\tau}{1-\gamma_\tau(\theta)}, \tag{LE-Q}$$

where $\gamma_\tau(\theta)$ is the Lagrange multiplier on the limited enforcement constraint.

[49] Theorem 4' offers the results for fully persistent types in an infinite horizon model.

[50] With positive selection, informational rents given to middle-types decrease, as the distribution is shifting towards higher-types $F_\tau(\theta) > F_{\tau+1}(\theta)$. In order to incentivize the highest types still active, middle-types will be distorted downwards in the future. Marginal unit prices are given by $p(q(\theta)) = c + (1 - F_\tau(\theta)/f_\tau(\theta)$ (Armstrong, 2016), which will be generally larger for each θ, and as such, average price will be larger at each q.

Without the need of an interim individual rationality constraint, the limited enforcement constraint generates dynamics, features an *initial phase*, in which quantities are set to zero for all types, for which $\gamma_0(\theta) = 1$, and a stationary *mature phase*, in which $\gamma_\tau(\theta) = X(\theta)$. More patient buyers, those with smaller $X(\theta)$, are closer to their first-best. Additionally, all else equal, higher types receive higher quantities.

The enforcement constraints are always binding, and the optimal tariffs are set as follows:

$$t_\tau^{le}(\theta) = \delta(\theta)\theta v(q_{\tau+1}^{le}), \tag{LE-T}$$

so tariffs are constant over time, but prices decrease between the *initial* and *mature* phase. Prices may vary by type, but in a simple CES model, prices are constant across types.

Therefore, limited enforcement generates backloading. This backloading is not a result of unequal discount rates, as it also appears in cases without exit. The intuition is that limited enforcement constraints create an asymmetry: the buyer compares current tariffs to future returns, so *ceteris paribus*, there is an incentive to minimize current quantities to maximize current profits. However, trade converges to the mature phase almost immediately, by the second period.

By including the additional interim individual rationality constraint ($u_\tau(\theta) \geq 0$), the initial phase lengthens. The reason is that the additional limited liability constraint forces quantity changes between periods to be smaller. The length of the initial path is dependent on the parameters for the buyer's return function and the discount factor. The higher the common discount factor or the lower the exit rate (the more patient the buyer), the longer the path before the mature phase. Similarly, the more responsive the return function, the longer the path. Though the path to convergence is longer, under a CES model, prices are constant across types within a given period.

OA-4.4 A Two-Type Illustrative Example

The purpose of this example is four-fold. First, I illustrate how the introduction of the limited enforcement constraint may distort quantities relative to perfect enforcement. Second, I show that lower types unambiguously reap higher net returns due to the enforcement constraint. The introduction of the enforcement constraints effectively raises their reservation return to participate in trade, forcing the seller to offer larger net return values to lower types. Third, I demonstrate that the optimal contract must be non-stationary. Fourth, I show through a solved example that the optimal stationary contract features *backloading*: unit prices decrease while quantities increase as relationships age.

OA-4.4.1 *Buyer's Types*

A buyer type-θ gains a gross return θq^β from q units of the product sold by the seller. Assume there are positive, yet diminishing marginal returns, i.e., $\beta \in (0,1)$. The buyer types can take values $\{\theta_L, \theta_H\}$, such that $\theta_L < \theta_H$. Let f_L (resp. f_H) be the probability that buyer is type L (resp. type H) and assume no exit, i.e., $X(\theta) = 0$.

OA-4.4.2 *A Stationary Contract*

For now, consider the optimal *stationary* contract. The optimal choice gives the buyer the net return $R(\theta_i) = \theta_i q_i^\beta - T(q_i)$. The seller designs the scheme to maximize:

$$max_{\{T_i,q_i\}} f_L(T_L - cq_L) + (1 - f_L)(T_H - cq_H)$$

where $T_i \equiv T(q_i)$, subject to incentive-compatibility constraints:

$$R(\theta_H) \equiv \theta_H q_H^{\beta} - T_H \geq \theta_H q_L^{\beta} - T_L, \tag{IC-H}$$

$$R(\theta_L) \equiv \theta_L q_L^{\beta} - T_L \geq \theta_L q_H^{\beta} - T_H. \tag{IC-L}$$

as well as the limited enforcement constraint:

$$\frac{\delta}{1-\delta}(R(\theta_i)) \geq T_i \quad i = L, H. \tag{LE-i}$$

This last constraint effectively (weakly) raises the minimum net rent that each buyer needs to obtain to participate in trade. The usual nonlinear pricing problem only requires that $R(\theta_i) \geq 0$. Instead, the limited enforcement case requires that $R(\theta_i) \geq (1-\delta)/\delta T_i > 0$, where the minimum return is endogenously determined. Notice that as $\delta \to 1$, the limiting case becomes the standard nonlinear pricing problem.[51]

To simplify the problem, assume that the IC-L and LE-H are slack while IC-H and LE-L are binding.[52] By using these assumptions on the constraints, one can obtain the optimal quantity allocations:

$$q_H^* = \left(\frac{\beta}{c}\theta_H\right)^{\frac{1}{1-\beta}},$$
$$q_L^* = \left(\frac{\beta}{c}\left[\theta_L - \frac{(1-\delta)\theta_L}{f_L} - \frac{(1-f_L)(\theta_H - \theta_L)}{f_L}\right]\right)^{\frac{1}{1-\beta}},$$

and optimal tariffs:

$$T_H^* = \theta_H q_H^{\beta} + (\delta\theta_L - \theta_H) q_L^{\beta},$$
$$T_L^* = \delta\theta_L q_L^{\beta}.$$

The tariffs are similar to that in the standard case, with the exception that the discount factor now enters the terms multiplying θ_L. Therefore, for a given quantity, tariffs are lower for both types.

The program's solution implies there is no distortion in quantities for type-H, as they purchase at the first-best level. However, type-L's purchases are shifted downwards. First, as is common in adverse selection problems, their purchases are distorted downwards to incentivize the revelation of type-H.

Second, contrary to the standard problem, extracting all rents from type-L is no longer feasible, as type-L would default. This generates a second downward pressure for quantities, as the standard quantity allocation for θ_L (i.e., when $\delta = 1$), together with the optimal tariffs for L under limited enforcement do not satisfy IC-H. To see this, notice that as IC-H was binding in the standard problem, type-H was on the margin between their standard bundle and the standard bundle for type-L. Thus, if the limited enforcement bundle for type-L keeps quantities

[51]The theoretical result that the buyer benefits from a deterioration of enforcement was previously discussed by Genicot and Ray (2006). In their model, they find that if better enforcement brings with it the deterioration of outside options and the seller has the bargaining power, the buyer will see their expected payoff increase. The opposite holds when the buyer has the bargaining power.

[52]All slack constraints are verified for the numerical example discussed below.

fixed (relative to the standard menu) and at the same time asks for lower tariffs, type-H buyers would now prefer the menu intended for type-L. As a result, the seller needs to reduce type-L's allocation, even further than would be required under the standard adverse selection problem.

OA-4.4.3 *Non-Stationarity*

Relative to the standard problem, the seller now needs to offer positive net returns to all buyers, in order to prevent default. Contrary to the results in Baron and Besanko (1984), the stationary contract is no longer the optimal contract. Instead, the seller could offer a dynamic contract with intertemporal incentives that use the promise of future returns to the buyer to discipline their behavior now. Through this approach, the seller can extract higher shares of surplus early on than would be feasible under a stationary contract, increasing their present-value lifetime profits.

The exact dynamic path depends on the return function and distribution of types of the buyer, as well as the marginal cost of the seller and the common discount factor. For that reason, I consider next a solved numerical example.

OA-4.4.4 *A Visual Example*

To visualize the problem, I consider a numerical example with the following values for the parameters: $\beta = 0.5$, $c = 1$, $f_L = 0.95$, $\theta_L = 1$, $\theta_H = 3$, $\delta = 0.9$.[53] Besides the incentive compatibility constraint and the limited enforcement constraint, I have also included the interim individual rationality constraint.

Online Appendix Figure OA-10 shows the levels of quantities, prices, profits per buyer, and buyer's net return for the example discussed above for different regimes: stationary with perfect enforcement (Baron-Myerson), stationary with limited enforcement, and dynamic with limited enforcement.

In solid green, the figure shows the allocation for type-H. As mentioned above, limited enforcement of contracts does not distort their consumption relative to perfect enforcement. In solid blue, the figure shows the allocation for type-L under perfect enforcement. Type-L receives lower quantities and higher prices than type-H and receives zero net return. In dashed-dot blue, the figure shows the stationary allocation for type-L under limited enforcement. Relative to perfect enforcement, type-L sees a reduction in quantities and an increase in net return, in line with the logic explained above. Importantly, as the buyer's return function features diminishing returns in q, lower levels of quantity for lower values of δ also imply the seller can charge *higher* unit prices to type-L.

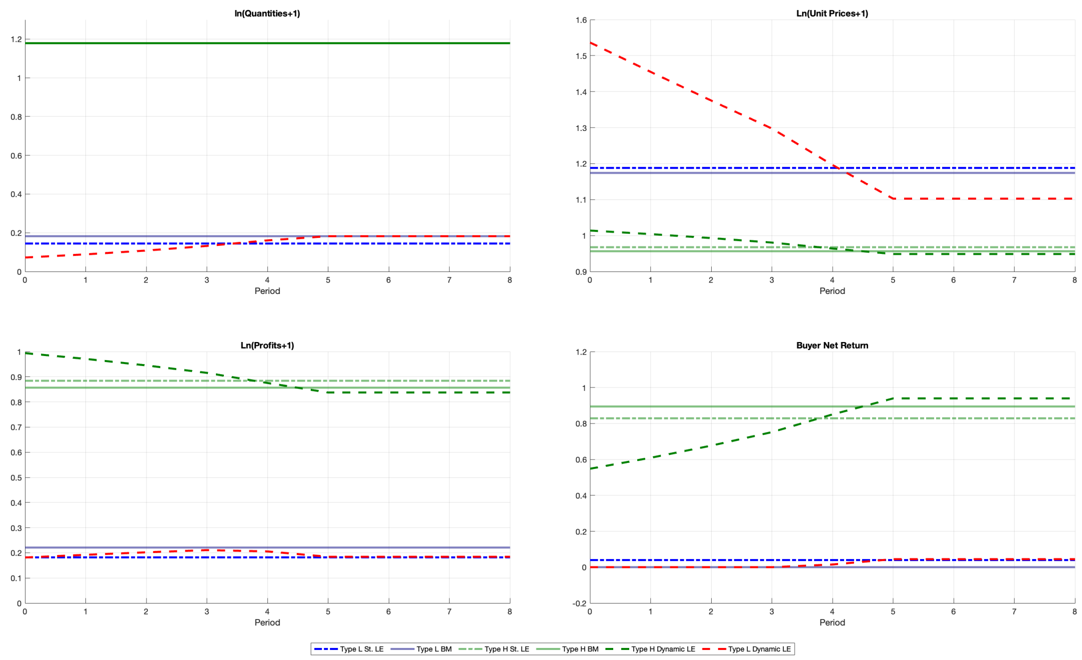
Lastly, the figure shows the optimal non-stationary path of prices and quantities in the dashed lines (red for type-L and green for type-H). The optimal path features *backloading* as quantities (weakly) increase and unit prices (weakly) decrease over time. As shown in the figure, this path of prices and quantities increases short-term expected profits from *each* buyer relative to the optimal stationary contract. Thus, the dynamics allow the seller to extract higher short-term profits for the high type as well. Indeed, in this example, the lifetime total profit in the dynamic case is 91% the level of the Baron-Myerson profit levels, whereas the stationary equilibrium reaches 88%. The seller can effectively prevent default now and increase present-value lifetime profits by offering higher surplus levels to the buyers in the future.

[53] The higher the difference between types, the higher the discount factor, the higher the elasticity β, or the bigger the share of high types, the longer the path to convergence.

Interestingly, the optimal path in the solved example features consumption for type-L in the long run that is greater than the stationary contracts with limited enforcement, as it converges to the Baron-Myerson allocation. Thus, in this case, the dynamics increase the long-term efficiency of the contracts.

In any case, the example shows that through the interaction market power on the seller side (which is reflected in the ability to offer incentive-compatible profit-maximizing menus) and the limited enforcement constraint, long-term contracts may display dynamics in which average quantities increase and unit prices decrease over time. Moreover, at any point in time, types consuming higher levels of quantities also enjoy lower unit prices. That is, this model of price discrimination with limited enforcement of contracts features i) *backloading* of prices and quantities, and ii) *quantity discounts* at any point in time.

Figure OA-10: **Example - Nonlinear Pricing and Limited Enforcement**



Notes: This figure shows Quantities, Prices, Profits, and Buyer Net Return for different enforcement and contract regimes. In dash-dot green, the optimal stationary contract for type-H under limited enforcement. In dashed green, the optimal dynamic contract for type-H under limited enforcement. In solid green, the optimal stationary contract for type-H under perfect enforcement. In solid blue, the optimal stationary contract for type-L under perfect enforcement. In dash-dot blue, the optimal stationary contract for type-L under limited enforcement. In dashed red, the optimal dynamic contract for type-H under limited enforcement. The parameters used in the example are: $\{\beta = 0.5, c = 1, f_L = 0.95, \theta_L = 1, \theta_H = 3, \delta = 0.9\}$.

OA-5 Proof of Lemma 1: $\Gamma_\tau(\overline{\theta}) = 1$

I prove that $\Gamma_\tau(\overline{\theta}) = 1$ for all τ. To begin, recall we assumed the outside option $\overline{u}_\tau(\theta)$ was equal to zero for all τ and all θ. Suppose instead that at some period k, the outside option is uniformly shifted downward by $\varepsilon > 0$ for all θ, that is, $\overline{u}_k(\theta) = -\varepsilon$. The enforcement constraint at k is now given by:

$$\delta[\sum_{s=1}^{\infty}\delta^{s-1}u_{k+s}(\theta)]-\bar{u}_k(\theta)=\sum_{s=1}^{\infty}\delta^s u_{k+s}(\theta)+\varepsilon\geq t_k(\theta)=\theta v(q_k(\theta))-u_k(\theta). \tag{52}$$

The seller's problem in the Lagrangian-form is

$$W(\varepsilon)=\max_{\{q_\tau(\theta),u_\tau(\theta)\}}\sum_{\tau=0}^{\infty}\delta^\tau\Big\{\int_{\underline{\theta}}^{\overline{\theta}}[\theta v(q_\tau(\theta))-cq_\tau(\theta)-u_\tau(\theta)]f(\theta)d\theta+ \tag{53}$$

$$\int_{\underline{\theta}}^{\overline{\theta}}[\sum_{s=1}^{\infty}\delta^s u_{\tau+s}+\varepsilon\times 1\{\tau=k\}-t_\tau(\theta)]d\Gamma_\tau(\theta)\Big\} \tag{54}$$

such that $u_\tau'(\theta)=\theta v'(q_\tau(\theta))$ for all τ,θ. The change in the value of the seller's problem given the uniform change in outside options is:

$$\frac{dW(\varepsilon)}{d\varepsilon}=\delta^k\int_{\underline{\theta}}^{\overline{\theta}}d\Gamma_k(\theta), \tag{55}$$

where the integral is the cumulative multiplier.

I argue that the quantities that solve the original problem still maximize the current one but that the tariffs are all shifted upward by the constant ε. That is, if $q_\tau(\theta)$ is the solution for the problem with $\bar{u}_\tau(\theta)=0$ for all θ and all τ with associated $t_\tau(\theta)$, $q_\tau(\theta)$ is also the solution for the problem with outside options $\bar{u}_\tau(\theta)=-\varepsilon\times 1\{\tau=k\}$ for all θ and all τ with associated tariffs equal to $t_\tau(\theta)+\varepsilon\times 1\{\tau=k\}$. The value of the problem for the seller is:

$$W(\varepsilon)=\sum_{\tau=0}^{\infty}\delta^\tau\Big\{\int_{\underline{\theta}}^{\overline{\theta}}[t_\tau(\theta)+\varepsilon\times 1\{\tau=k\}-cq_\tau(\theta)]f(\theta)d\theta\Big\} \tag{56}$$

$$=\sum_{\tau=0}^{\infty}\delta^\tau\Big\{\int_{\underline{\theta}}^{\overline{\theta}}[t_\tau(\theta)-cq_\tau(\theta)]f(\theta)d\theta\Big\}+\delta^k\varepsilon. \tag{57}$$

So

$$\frac{dW(\varepsilon)}{d\varepsilon}=\delta^k. \tag{58}$$

Therefore, from equations 55 and 58, the cumulative multiplier for any k will satisfy the following property:

$$\Gamma_k(\overline{\theta})\equiv\int_{\underline{\theta}}^{\overline{\theta}}d\Gamma_k(\theta)=\frac{dW(\varepsilon)}{d\varepsilon}\frac{1}{\delta^k}=1. \tag{59}$$

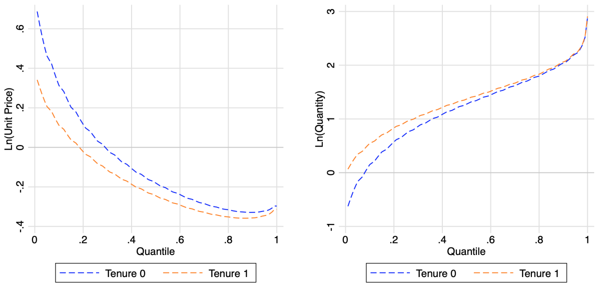
OA-6 Monte Carlo Study

The Monte Carlo studies the behavior of my estimators for two periods of a dynamic contract without breakups. I use the following design. The return function is $v(\theta,q)=\theta q^{1/2}$. The type distribution is Weibull with scale parameter equal to 1 and shape parameter equal to 2, $F(\theta)=1-exp(-(\theta-1)^k)$, normalized so $\underline{\theta}=1$.[54] Marginal cost is 0.45. Although

[54]Recall that the model requires the type distribution to verify the monotone hazard rate condition, $\frac{d}{d\theta}\frac{F(\theta)}{f(\theta)}\geq 0\geq\frac{d}{d\theta}\frac{1-F(\theta)}{f(\theta)}$. Distributions that satisfy the monotone hazard rate condition include: Uniform, Normal, Logistic,

Figure OA-11: **Prices and Quantities by Quantile**

(a) **Unit Prices** (b) **Quantity**



Notes: These figures show the level of prices and quantities by quantile of quantity for tenure 0 and tenure 1 in the Monte Carlo simulation.

the multiplier function $\Gamma_\tau(\theta)$ is the solution to a differential equation linking the type distribution $F(\theta)$, the marginal cost, and the average base marginal return of types $\tilde{\theta} \leq \theta$, I parametrize it as a logistic distribution. In tenure 0, $\Gamma_0(\theta)$ has location parameter equal to 1 and scale parameter equal to 0.5. Instead, in tenure 1, $\Gamma_1(\theta)$ has location parameter 1 and scale 0.35. The lower scale parameter at tenure 1 reflects the idea that over time, the limited enforcement constraint is less binding. I construct the tariffs following Pavan et al. (2014): $t_\tau(\theta) = \theta q_\tau(\theta)^{1/2} - \int_{\underline{\theta}}^{\theta} q_\tau(x)^{1/2} dx$.

I randomly draw 1000 values of θ using $F(\theta)$ and obtain corresponding quantities $q_0(\theta)$ and $q_1(\theta)$ using the first-order condition of the seller and the assumed parametrizations of the return function, marginal cost, and multiplier at tenure 0 and 1. Then, I obtain the corresponding tariffs and I apply my estimator as defined in the previous sections to estimate $\{\theta, U(\cdot), \Gamma_\tau(\cdot)\}$. I repeat this 300 times to construct the dispersion for my estimates.

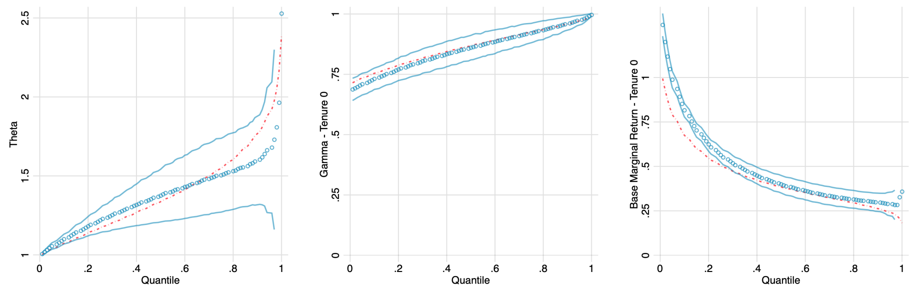
Online Appendix Figure OA-11 shows the (log) average prices and average quantities generated by the model for the two types of tenure. The model delivers quantity discounts (decreasing unit prices in θ), strict mononoticity of quantity (increasing quantities in θ), and backloading in the dynamic model, namely, further discounts and larger quantities in tenure 1 for each θ.

Online Appendix Figure OA-12 shows the results of the estimated Gamma distribution and the base marginal return, again in blue the estimated results and in red the true values. Both cases indicate good fit. Subfigure (a) shows the estimated $\hat{\theta}$ in blue and true θ in red by quantile. Dispersion at the 95 percent level are included for all except the top 2 quantiles, as they start to diverge. Overall, the figure shows a good fit, with most sections of including the true θ within their dispersion.

Next, I show the tenure 1's results estimates. Recall that the first-order condition of the seller now includes a backward-looking variable $1 - \Gamma_0(\theta)$ that keeps track of whether the limited commitment constraint was binding in the past. This variable is used by seller as a promise-keeping constraint that guarantees the seller delivers higher quantities and return in the future to prevent buyers from defaulting in the past. In my estimation, I use the tenure 0's predicted $\widehat{\Gamma}_0(\theta(\alpha))$ for each quantile α. Online Appendix Figure OA-13 shows the estimated Gamma distribution and the base marginal return. Although the fit is worse than in tenure 0, the dispersion of both gamma and the base marginal return include tend to include their true values.

Extreme Value (including Frechet), Weibull (shape parameter ≥ 1), Exponential, and Power functions.

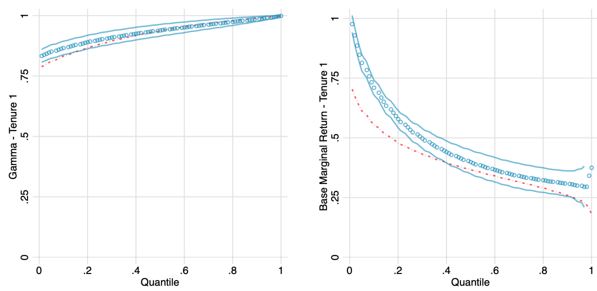
Figure OA-12: **Monte Carlo Results for Tenure 0**



(a) Types θ (b) Γ_0 Distribution (c) $v'(\cdot)$

Notes: Panel (a) plots the true (red) and estimated distribution of types (in blue) by quantile of quantity. Panel (b) plots the true (red) and estimated value (blue) of the LE multiplier for tenure 0 by quantile of quantity. Panel (c) plots the true (red) and estimated value (blue) of the base marginal return for tenure 0 by quantile of quantity. Error margins indicate ± 1.96 variation around estimated mean from 300 simulations.

Figure OA-13: **Monte Carlo Results for Tenure 1**



(a) Γ_1 Distribution (b) $v'(\cdot)$

Notes: Panel (a) plots the true (red) and estimated value (blue) of the LE multiplier for tenure 1 by quantile of quantity, with error margins indicating ± 1.96 variation around the estimated mean. Panel (b) plots the true (red) and estimated value (blue) of the base marginal return for tenure 1 by quantile of quantity, with error margins indicating ± 1.96 variation around the estimated mean from 300 simulations.

With respect to the differences between true and estimated functions, I find that the slight upward bias in the Gamma function for tenure 1 disappears if I use the true $\Gamma_0(\theta)$ function instead of the estimated $\widehat{\Gamma}_0$, suggesting that the bias is generated by sampling error in the tenure 0 estimates. Moreover, differences in the base marginal return for both tenure 0 and tenure 1 come from approximating the tariff function as log-linear. In the Monte-Carlo, the change in unit price is very steep for low-types, and this generates some approximation error for low-types in terms of the base marginal return function. Despite this error, the coefficient of the base return function is correctly estimated when using the assumed parametrization, observations of quantity, and the nonparametric estimates of $v'(\cdot)$ as target. In particular, the estimated coefficient cannot be rejected to be different from 0.5 (the assumed value in simulation).

OA-7 Evidence for Marginal Costs Constancy Assumption

I provide empirical support for the assumption of constant marginal cost in three ways.

First, I present evidence that average variable cost (AVC) is relatively constant over time.

For each seller i at time t, I construct *quarterly* measures of average cost by dividing total variable cost (intermediate inputs plus labor) in the quarter by total quantity sold in the quarter:

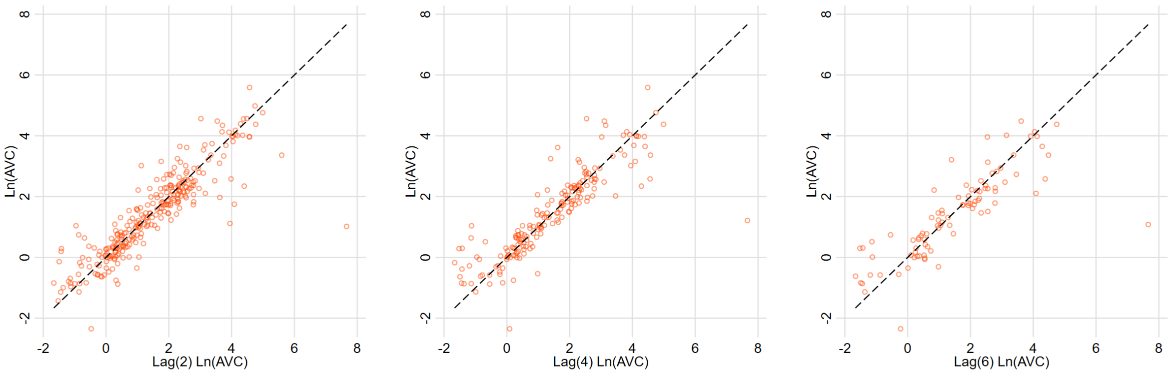
$$AVC_{it} = \frac{VC_i(Q_{it})}{Q_{it}},$$

where $VC_i(\cdot)$ is the variable cost function. Marginal cost is related to the previous equation via the derivative of the variable cost function: $MC_i(Q_{it}) = VC_i'(Q_{it})$. If marginal cost is constant, then $VC_i'(Q_{it}) = c_i Q_{it}$ and $AVC_{it} = c_i$. Therefore, strong serial correlation in AVC across periods indicates the following relationship:

$$AVC_{it} = c_i + \varepsilon_{it}.$$

Appendix Figure OA-14 presents a scatter plot of the (log) average variable cost on two, four, and six lags, with the dashed diagonal presenting a 1-to-1 fit. The figure shows that even after one and a half years apart, the average variable cost traces the diagonal fairly well.[55] This type of test is meaningful as sellers do experience variation in sales across months (Online Appendix Table OA-10), and therefore Q_{it} is non-constant.

Figure OA-14: **Serial Correlation**



(a) 2 Lags (b) 4 Lags (c) 6 Lags

Notes: These figures present the scatter plots of firm-level quarterly measures of average variable costs against 2 quarter lags (a), 4 quarters (b) and 6 quarters (c).

Second, I verify the constancy of average variable costs using a regression framework by regressing (log) average variable costs on seller fixed effects. I find that seller effects explain 87% of all variation using quarterly data and 84% using monthly data.

Third, under the assumption of constant marginal costs, we obtain the following accounting relationship for total variable costs: $VC_{it} = c_i Q_{it}$. Taking logs yields:

$$\ln(VC_{it}) = \ln(c_i) + \ln(Q_{it}).$$

This equation creates a testable framework for regression:

$$\ln(VC_{it}) = \beta_Q^c \ln(Q_{it}) + \ln(c_i) + \varepsilon_{it},$$

where $\beta_Q^c = 1$ under constant marginal costs, $\ln(c_i)$ is captured by a seller fixed effect, and ε_{it}

[55] A similar relationship exists if we focus only on monthly variation.

Table OA-11: Test for constancy of marginal cost

VARIABLES	(1) ln(VC)	(2) ln(VC)
ln(Q)	0.163**	0.757**
	(0.0723)	(0.302)
P-Value ($\beta_Q^c = 1$)	0.000	0.415
Observations	384	384
Seller FE	Yes	Yes
Time	Quarterly	Quarterly
Method	OLS	IV

Notes: This table presents the results of the test for constancy of marginal costs, of (log) total variables costs on (log) quantity. Column(1) reports OLS and Column (2) reports the instrumental variable results. Unit of observation is at the seller-quarter-level. Standard errors are clustered at the seller level. ***p<0.01, **p<0.05, *p<0.1

is noise, possibly stemming from model specification (i.e., true costs are non-constant and thus c_{it} is time-varying).

Notice that an OLS regression would not serve to test this equation if true marginal costs are time-varying, even if they are constant at the output level within the time period. An increase in true time-varying marginal cost is likely associated with a total decrease in quantity sold (as the seller increases prices to buyers). Thus, as quantity increases total variable costs, $\beta_Q^c > 0$, the negative relationship between costs and observed quantities implies downward bias in OLS due to omitted variable bias.

For that reason, I test this equation using an instrumental variable approach that exogenously shifts Q_{it} from changes in marginal costs captured by ε_{it}. Specifically, I use downstream demand shift-share style shocks in the spirit of Acemoglu et al. (2016) and Huneeus (2018). For a given selling firm i, I consider their 2015 demand share s_{ij}^{2015} over buyers j. Then, for each buyer, I regress their quarterly volume of log sales on buyer fixed effects and quarter-year fixed effects and collect the residuals as demand shocks $shock_{jt}^d$. For each seller, I obtain the weighted average of their exposure to potential demand shocks IV_{it}^d as follows:

$$IV_{it}^d = \sum_j s_{ij}^{2015} \times shock_{jt}^d.$$

I then run a regression for the testing equation using quarterly data at the seller level, using IV_{it}^d as an instrument for quarterly quantity Q_{it}.

Internet Appendix Table OA-11 shows the results. First, OLS (Column 1) shows a downward bias relative to the IV (Column 2), indicating some degree of model misspecification or measurement error in total quantity. Second, in the instrumental variable approach, we fail to reject that β_Q^c is equal to 1 (although, the point estimate is not precisely estimated at 1). Therefore, the test is again consistent with a constant marginal cost assumption.

Thus, all in all, the constant marginal cost assumption is not incredibly restrictive in this setting.

OA-8 Additional Estimation Results and Model Fit

OA-8.0.1 *Tariff Function*

Despite the simple approximation of the tariff function in equation 5, the within-tenure seller-specific tariff functions show a good fit. The average R-squared is close to 0.80, and the distribution of R-squared estimates for each seller-tenure (Figure OA-15) shows a good fit across the board.

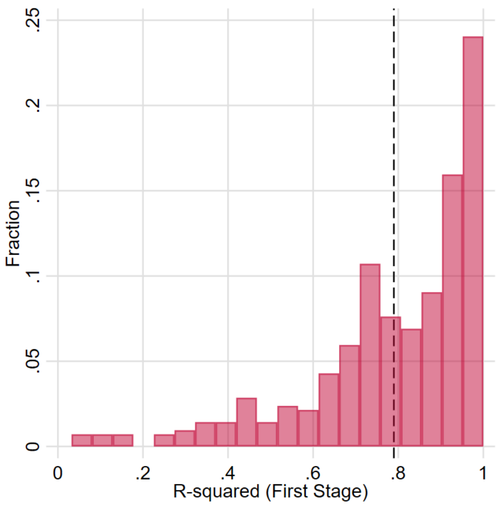


Figure OA-15: R-squared Distribution in the Estimation of the Tariff Function

Notes: This figure presents the distribution of R-squared values from seller-tenure-year regressions (equation 5).

Of course, as the fit is not perfect, it is worth highlighting some sources of measurement error in the tariff function. First, it is possible that the firm price schedule has higher-order terms, which would generate measurement error. However, this concern is small, as estimating a quadratic model only improves the R-squared on average by 0.008. Second, it is possible that, besides pricing on tenure and quantity, the firm is also pricing based on other unobservable characteristics (to the econometrician), which creates misspecification error, translating into measurement error. This would be particularly worrisome if the price schedule over quantities and tenure is not linearly separable from the other pricing characteristics. However, as shown in Table OA-6, the coefficients for prices on quantities and tenure are unaffected by the inclusion of a large set of buyer characteristics, supporting the assertion that pricing on other (plausibly unobserved) characteristics might enter as orthogonal measurement error.

OA-8.1 Survival Function Probability

Online Appendix Figure OA-16 presents estimated survival probabilities by age of relationship and quantile of quantity, with variation representing differences across seller-years.

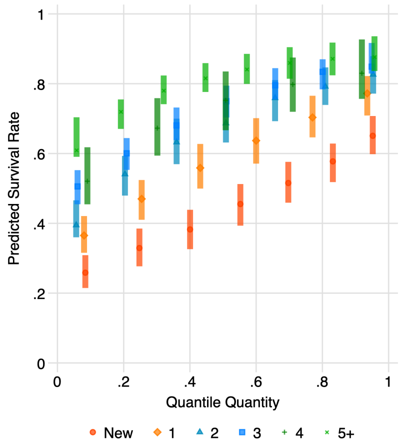


Figure OA-16: Survival Probability Function

Notes: This figure presents the estimated survival probability by quantile of quantity and age of relationship across seller-years. Confidence intervals represent the 90% level of variation across sellers, with standard errors clustered at the seller-year level.

OA-8.2 Distribution of t-Statistics against Standard Model Null

Online Appendix Table OA-12 shows the distribution of t-statistics for tests against a standard model null.

Table OA-12: **Distribution of t-Statistics**

	p10	p25	p50	p75	p90
Tenure 0	0.31	4.64	11.55	30.08	109.27

Notes: This table reports distribution of t-statistics for tests against a standard model null (e.g., $\Gamma_0(\cdot) = 1$).

OA-8.3 Parametrization of the Base Return Function

To conduct counterfactual experiments that consider quantities beyond those observed in the data, I parametrize the seller-specific buyer's return function $v(q) = kq^{\beta}$ for $k > 0$ and $\beta \in (0,1)$. This return function satisfies the modeling assumptions $v'(\cdot) > 0$ and $v''(\cdot) < 0$.

To estimate the parameters, I consider tenure 0 transactions between buyer i and the seller at a given year and perform the following linear least squares regression:

$$\ln(\widehat{v'}_i) = \ln(k\beta) + (\beta - 1)\ln(q_i) + \varepsilon_i,$$

using $v'(q) = k\beta q^{\beta-1}$, the estimated base marginal returns $\widehat{v'}_i$, and under the assumption that ε_i is Gaussian error.

Online Appendix Table OA-13 presents the distribution of k and β.

Table OA-13: **Parameters of Return Function**

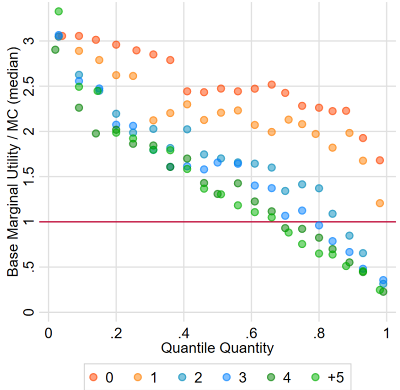
	mean	p10	p25	p50	p75	p90
β	0.56	0.30	0.48	0.61	0.76	0.82
k	171.23	9.00	17.24	39.64	86.61	282.40

Notes: This table reports distribution of estimated values for the ex-post parametrization of the return function.

OA-8.4 Economic Magnitudes: Base Marginal Return

Online Appendix Figure OA-17 presents a binscatter of the ratio of marginal revenue product (base marginal return) over marginal costs against the quantile of quantity, across sellers for tenure 0. It shows that the return of the input for the buyer is greater than the private marginal cost of providing it for the seller, for a majority of the buyers. For instance, the median buyer obtains 1.5 dollars of revenue for each dollar spent by the seller to produce the product.

Figure OA-17: **Base Marginal Return over Marginal Costs**



Notes: This figure plots the median of the ratio of base marginal return to marginal costs across sellers by quantile of quantity for each tenure.

OA-8.5 Model Fit

Online Appendix Figure OA-18 presents the statistical fit of the model across tenures. It plots a reordered equation I-EQ's left-hand side on the X-axis and the model's prediction using estimated coefficients of the right-hand side on the Y-axis.[56] Fit generally worsens for higher tenures; the results from Monte Carlo studies in Online Appendix OA-6 suggest that the decrease in statistical fit is driven by noise from using estimates for limited enforcement multipliers $\Gamma_s(\cdot)$ for earlier tenures s.

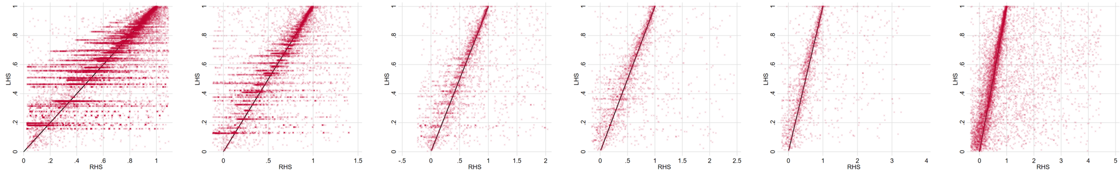
Online Appendix Figure OA-19 shows the fit in terms of quantities. To obtain quantities, I use the parametrization $v(q) = kq^{\beta}$, for $k > 0$ and $\beta \in (0,1)$ and the closed-form formula in 3.

[56]Reorder equation I-EQ to obtain:

$$\alpha = \Gamma_\tau(\alpha) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\alpha)) - \left[\frac{T'_\tau(q_\tau(\alpha)) - c_\tau}{T'_\tau(q_\tau(\alpha))} - \gamma_\tau(\alpha)\right]\frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)},$$

and use the estimated analogues of the right-hand side to make the predictions.

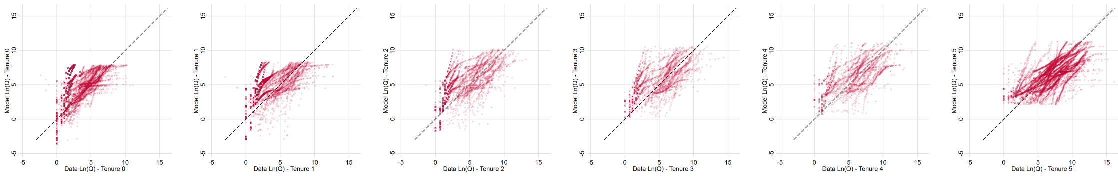
Figure OA-18: **Model Fit - Statistical**



(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

Notes: These figures show binscatters of statistical fit of the model across tenures as implied by identification equation I-EQ. On the X-axis, it shows the predicted cumulative distribution function for the observation while on the Y-axis it plots the observed value.

Figure OA-19: **Model Fit - Quantities**

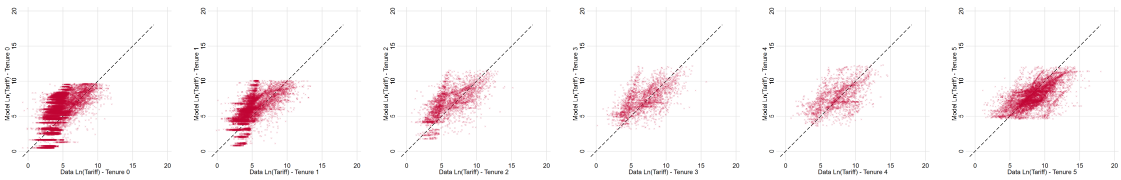


(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

Notes: These figures display binscatters of model fit according to quantities. Estimated quantities use the close-form formula under the CES parametrization of the return function, as discussed in Appendix OA-13. The X-axis plots the observed (log) quantities and Y-axis model predicted (log) quantities.

Online Appendix Figure OA-20 shows the fit of tariffs. To generate tariffs in the model, I use the empirical equivalent of equation t-RULE.

Figure OA-20: **Model Fit - Tariffs**

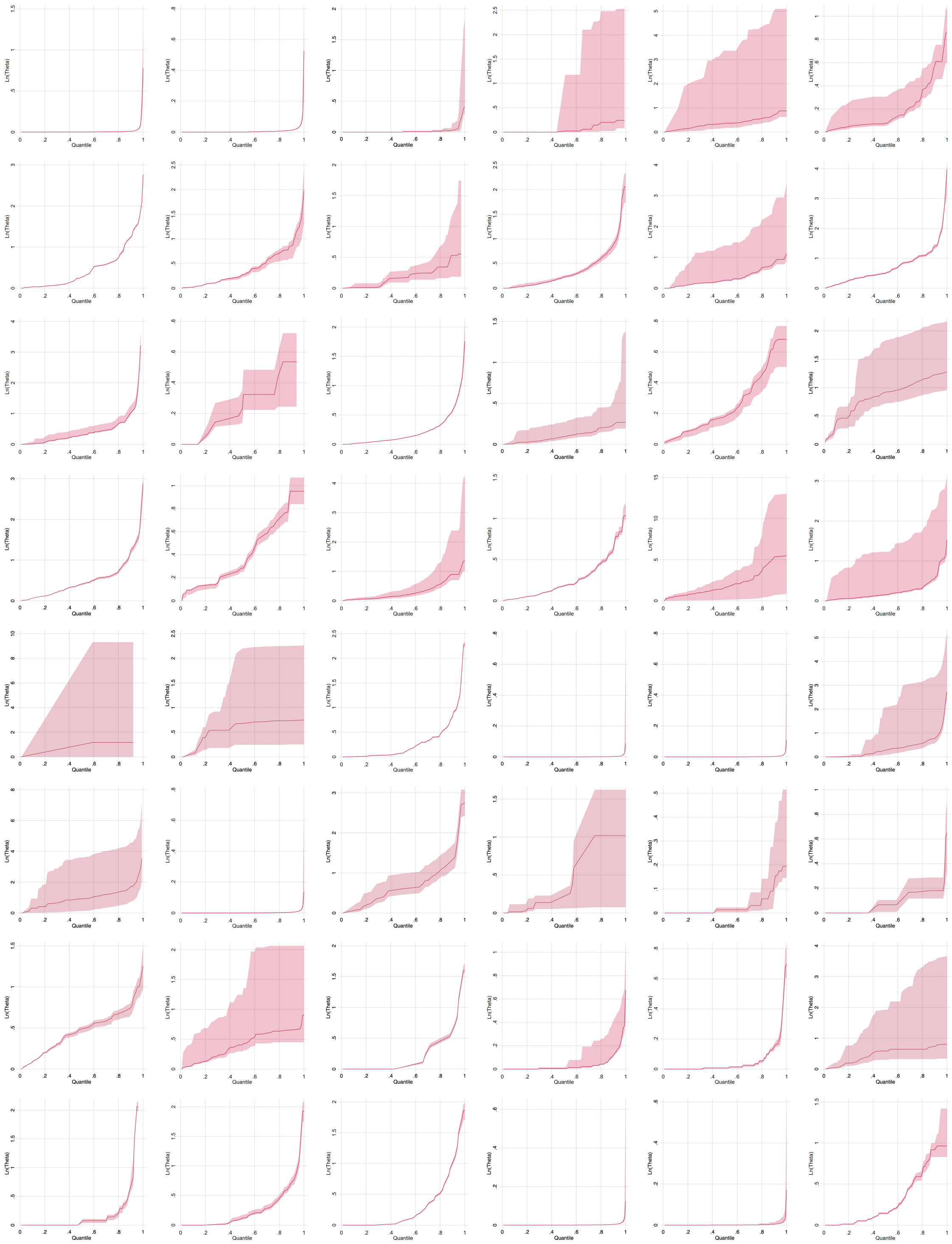


(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

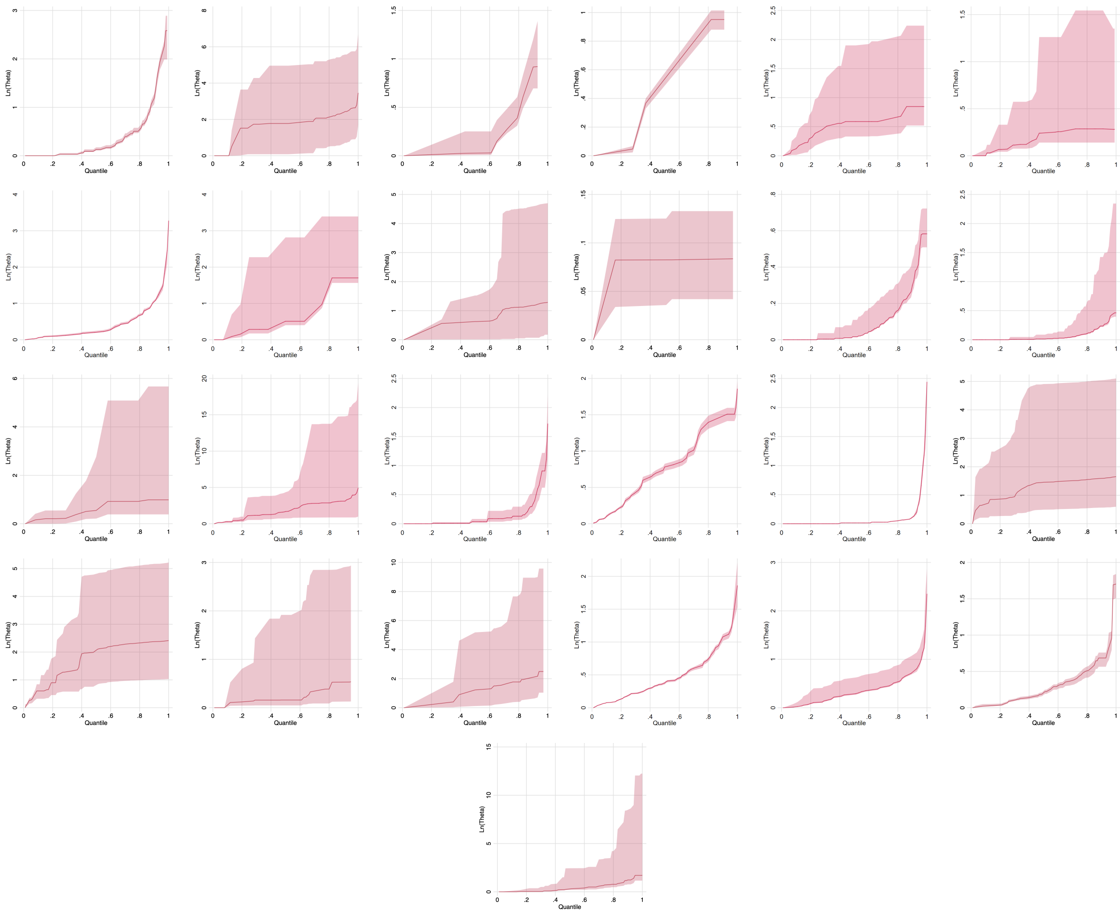
Notes: These figures display binscatters of model fit according to tariffs. Estimated tariffs are generated by using the empirical analogue of the tariffs rule t-RULE, taking as inputs estimated parameters θ, the parametrized return function $v(\cdot)$, and model generated quantities. The X-axis plots the observed (log) tariffs and Y-axis model predicted (log) tariffs.

OA-8.6 Bootstrapped Distribution of Types

Figure OA-21: Bootstrapped Distribution of Types



Bootstrapped Distribution of Types (Continued)



Notes: This figure plots distribution of types (log type $ln(\theta)$ by quantile of quantity) for each seller-year. The error bars show variation at the 90% confidence interval level, obtained from 30 bootstrapped simulations for each seller-year.

OA-9 Additional Counterfactual Results

OA-9.1 Computation of Counterfactuals

Counterfactual (a). I compute quantities based on the distribution of estimated types at different tenures and the quantity allocation equation 3 with $\Gamma_\tau(\cdot)$ set to 1 and $\gamma_\tau(\cdot)$ set to 0. I also set $\Gamma_s(\cdot)$ to 1 for $s < \tau$. With quantities in hand, the tariffs are set to satisfy incentive compatibility using equation t-RULE.

Counterfactual (b). Under the assumed base return function, the optimal uniform price is $p^l = c/\beta$ for any quantity. The corresponding type θ's demand is given by $q^l(\theta) = (k\beta\theta/p^l)^{1/(1-\beta)}$. This stationary menu will be insufficient for some enforcement constraints. Given exogenous hazard rates $X(\theta)$, the stationary enforcement constraint will be given by:

$$\delta(1 - X(\theta)) \geq \beta, \tag{U-LE}$$

which indicates that the rate of return captured by β has to be smaller than the buyer-specific discount rate. Notice that this limited enforcement constraint will hold for any other uniform price, so buyers who are willing to default at the optimal uniform price p^l will also be willing to default at any other alternative uniform price p^l_a, including $p^l_a = c$, which would generally imply an efficient allocation.

Under a monotonicity assumption on $X(\theta)$,the seller will set a minimum quantity $\underline{q}^l$ that the buyer needs to announce in order to be served.[57] In particular, it will only serve $q(\theta) \geq \underline{q}^l$, where $\underline{q}^l = min\{q^l(\theta)|\delta(1-X(\theta)) \geq \beta\}$. In the counterfactual exercise, I set their quantities to zero to those θ with $q^l(\theta) < \underline{q}^l$.[58]

Counterfactual (c). Quantities and tariffs are those determined in Counterfactual 2. However, as buyers are precluded from the possibility of default, the seller serves all buyers. Thus, no quantity is set to zero.

OA-9.2 Results

This subsection presents comparisons of different counterfactual models relative to the baseline nonlinear pricing regime with limited enforcement. Online Appendix Table OA-14 shows all the results. The table present the *share of observations* in each percentile group for which each reported category (e.g., buyer's net return) is greater under the baseline than under the alternative. The main takeaways are the following.

Buyers. Small-quantity buyers tend to prefer limited enforcement of contracts over perfect enforcement. They can effectively use the threat of default to reap higher returns. In contrast, the median and top buyers prefer perfect enforcement in the short term but limited enforcement in the long term. Under weak enforcement of contracts, buyers prefer price discrimination over uniform pricing, as otherwise they would be excluded from trade (only median and top buyers prefer uniform pricing in the long term). However, if exclusion and default are restricted, most buyers prefer uniform pricing.

Sellers. Sellers prefer limited enforcement in the short term but perfect enforcement in the long term. Under weak enforcement of contracts, they enjoy the ability to price discriminate, as it allows them to sell to buyers that would otherwise be excluded from trade. In contrast, if

[57] The monotonicity on the hazard rate $X'(\theta) < 0$ is observed in the data.

[58] In this counterfactual exercise, I use an additional assumption: buyers demand truthfully the optimal level of quantity that is consistent with prices and full enforcement.

enforcement is strong, sellers prefer uniform pricing in the short term but price discrimination in the long term. This preference is driven by the rapid increase in quantities, despite the decrease in unit prices offered to most buyers as an incentive not to default.

Table OA-14: **Counterfactual Policies**

		Nonlinear + Perfect						**Uniform + Limited**						**Uniform + Perfect**					
		10%	25%	50%	75%	100%	Agg.	10%	25%	50%	75%	100%	Agg.	10%	25%	50%	75%	100%	Agg.
Buyer Return	*Tenure 0*	43.4	38.2	11.0	4.9	7.1	6.9	97.3	96.5	96.0	94.3	91.7	92.0	0.1	0.2	0.6	7.0	41.8	38.5
	Tenure 1	68.3	55.3	23.0	9.4	11.9	11.8	94.6	92.2	88.6	88.0	87.4	87.6	0.1	0.1	0.2	13.5	54.9	47.0
	Tenure 2	64.3	46.5	31.1	26.2	28.4	28.3	83.8	79.6	70.3	66.9	63.1	63.6	1.2	0.4	0.9	10.9	32.1	29.6
	Tenure 3	66.3	59.8	40.5	32.3	38.0	37.6	79.7	71.4	59.6	54.6	55.4	55.5	3.1	0.8	1.6	11.2	27.8	25.5
	Tenure 4	61.2	48.6	43.5	42.6	50.5	49.0	69.0	59.9	47.6	47.9	46.3	46.7	5.3	1.2	4.9	8.8	21.3	18.6
	Tenure 5	58.7	61.8	66.1	59.6	69.5	67.8	69.1	62.2	38.3	34.8	32.8	33.5	0.7	1.6	2.9	9.0	22.0	19.6
Seller Profit	*Tenure 0*	34.1	41.6	88.2	94.9	92.8	93.0	92.7	92.6	96.4	98.0	98.4	98.4	7.1	7.4	11.1	35.0	47.4	46.4
	Tenure 1	53.9	55.0	83.3	90.6	88.1	88.3	99.1	96.7	94.8	97.1	89.9	91.2	29.1	18.4	29.8	44.8	52.8	51.0
	Tenure 2	46.6	49.1	71.5	73.8	71.6	71.8	95.0	97.0	98.2	99.5	97.5	97.7	34.1	35.1	50.8	69.1	86.6	84.3
	Tenure 3	45.8	48.1	61.2	67.9	62.0	62.5	96.5	99.2	97.5	99.3	93.9	94.5	49.6	50.0	61.6	77.9	86.6	85.1
	Tenure 4	52.0	47.1	59.1	57.4	49.5	51.1	92.9	97.6	95.0	95.2	94.5	94.6	53.5	64.2	71.4	86.5	93.7	91.5
	Tenure 5	56.1	42.5	36.8	40.6	30.5	32.4	93.4	93.5	96.0	97.4	95.9	96.1	64.9	66.0	81.9	93.1	94.8	93.9
Surplus	*Tenure 0*	18.6	18.9	9.0	3.8	2.6	2.7	98.4	98.1	98.8	98.5	99.5	99.5	3.8	4.1	5.2	12.0	65.5	60.4
	Tenure 1	40.5	41.7	30.3	12.6	29.6	26.9	97.5	96.2	97.3	99.2	100.0	99.8	6.0	7.4	11.0	31.9	76.1	67.6
	Tenure 2	47.8	50.9	48.3	63.2	72.8	71.5	90.9	90.7	91.6	98.6	99.7	99.5	15.3	16.4	27.3	57.0	95.0	90.2
	Tenure 3	61.0	57.8	69.7	76.8	69.9	70.5	93.8	92.3	89.5	98.5	99.6	99.4	24.6	26.5	37.4	69.1	98.4	94.0
	Tenure 4	65.6	71.9	74.5	77.1	67.3	69.2	81.0	87.8	85.4	98.4	99.5	98.9	25.7	34.3	51.0	79.4	97.9	92.9
	Tenure 5	74.4	79.7	88.7	91.2	84.9	85.9	84.8	86.6	80.6	97.1	100.0	98.7	30.8	34.1	53.1	86.4	99.9	95.7
Unit Prices	*Tenure 0*	75.9	75.4	89.0	94.5	92.9	93.0	93.6	93.1	95.4	90.3	42.9	47.4	93.6	93.1	95.4	90.3	42.9	47.4
	Tenure 1	55.3	55.5	77.6	90.5	88.0	88.2	98.6	96.8	87.9	68.2	24.6	33.1	98.6	96.8	87.9	68.2	24.6	33.1
	Tenure 2	38.6	55.1	67.6	73.7	71.7	71.8	92.3	95.0	90.9	64.5	18.0	23.6	92.0	94.9	90.9	64.5	18.0	23.6
	Tenure 3	36.5	41.4	58.3	67.9	61.8	62.3	91.2	97.0	89.1	56.0	13.7	19.7	90.8	97.0	89.1	56.0	13.7	19.7
	Tenure 4	37.9	51.7	56.7	59.1	49.4	51.2	89.2	95.8	88.0	63.2	18.7	28.8	88.7	95.8	88.0	63.2	18.6	28.6
	Tenure 5	34.4	34.1	33.8	39.5	30.5	32.0	90.0	91.4	87.6	54.0	10.4	20.5	89.1	91.2	87.5	53.7	10.0	20.1
% Excluded	*Tenure 0*	-	-	-	-	-	-	97.3	96.4	95.8	94.1	90.5	90.9	-	-	-	-	-	-
	Tenure 1	-	-	-	-	-	-	93.4	91.9	88.6	87.3	85.8	86.1	-	-	-	-	-	-
	Tenure 2	-	-	-	-	-	-	81.5	77.8	70.1	65.7	61.3	61.9	-	-	-	-	-	-
	Tenure 3	-	-	-	-	-	-	76.9	69.0	59.5	51.5	50.0	50.4	-	-	-	-	-	-
	Tenure 4	-	-	-	-	-	-	66.8	58.1	47.5	44.7	43.5	50.0	-	-	-	-	-	-
	Tenure 5	-	-	-	-	-	-	65.3	58.8	37.5	29.8	25.4	26.7	-	-	-	-	-	-

Notes: This table reports the % share of observations for which the reported category (e.g., Buyer's Net Return) is greater under the observed nonlinear pricing regime than under the alternative policy. The values are reported across different tenures and percentile groups in the distribution of types. Percentile groups are defined based on quantiles as follows: the 10% group includes all buyers within seller-year-tenure quantiles from 0 to 10% (non-inclusive), the 25% group includes buyers within quantiles from 10% to 25% (non-inclusive), and this pattern continues for all other percentile groups. The policies considered are (a) Nonlinear pricing with perfect enforcement, (b) Uniform monopolist pricing with limited enforcement, and (c) Uniform monopolist pricing with perfect enforcement. The reported categories are Buyer's Net Return, Seller's Profits, Total Surplus, Unit Prices, and percentage of Excluded Buyers.