Take the Goods and Run: Contracting Frictions and Market Power in Supply Chains*

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This Version: May 27, 2022
First version: October 16, 2020

Abstract

Firms in developing countries often face concentrated input markets and contracting frictions. This paper studies the efficiency of self-enforced relational agreements, a common solution to contracting frictions, when sellers have significant market power and contracts cannot be enforced through courts. To this end, I develop a dynamic contracting model with limited enforcement in which buyers can default on their trade-credit debt without legal penalties. The model is shown to be identified and is estimated using a new transaction-level dataset from the Ecuadorian manufacturing supply chain. My key empirical finding is that bilateral trade is inefficiently low in early periods of the relationship, but converges toward efficiency over time, despite sellers’ market power. Counterfactual simulations imply that both market power and enforcement contribute to inefficiencies in trade, as addressing either friction alone leads to welfare losses, whereas relaxing both frictions can lead to significant efficiency gains.

∗Thanks to Jesse Shapiro, Rafael La Porta, and Neil Thakral for their continued guidance and support, as well as Lorenzo Aldeco, Dan Bjorkegren, Joaquin Blaum, Javier Brugués, Pedro Dal Bó, Marcel Fafchamps, Jack Fanning, Andrew Foster, John Friedman, Samuele Giambra, Stefan Hut, Amanda Loyola, Teddy Melkonnen, Bobby Pakzad-Hurson, Elena Pastorino, Diego Ramos-Toro, Pau Roldan-Blanco, Bryce Steinberg, Marcel Peruffo, Julia Tanndal, Marta Troya Martínez (discussant), Matt Turner, and members of audiences from multiple seminars for helpful conversations and comments. I thank Cristian Chicaiza, Alexiss Mejía, and María Eugenia Andrade at the Servicios de Rentas Internas of Ecuador for help with the data. Carolina Alvarez provided excellent research assistance. I gratefully acknowledge financial support from the Bank of Spain and the Nelson Center for Entrepreneurship at Brown University.

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1 Introduction

When courts cannot enforce contracts, trading partners often resort to long-term relational contracts, sustained through repeated interactions, to ease frictions and constrain opportunistic behavior (Johnson et al., 2002). As weak contract enforcement is a common feature of developing economies, relational agreements are highly relevant between-firm organizational structures. Understanding the efficiency of these informal agreements is essential for policy-makers in developing countries, as they frequently have to make trade-offs regarding where to focus their reform efforts.

The traditional view sees contracting frictions as a hindrance that distorts productive decisions (La Porta et al., 1997; Nunn, 2007), implying that, as a standard solution, relational contracts may be inefficient. Notably, however, the very same economies where enforcement constraints are likely to matter may also experience additional frictions, such as high market concentration, making them second-best environments (Rodrik, 2008). Under the presence of seller market power, weak enforcement may improve the buyer’s relative bargaining power, limiting downstream distortions while increasing the efficiency of a relationship relative to a perfect enforcement world (Genicot and Ray, 2006). Therefore, the efficiency of relational agreements remains unclear.

This paper uses theory and data to quantify the static (period-by-period) efficiency of self-enforced long-term relationships in the presence of seller market power and limited external enforcement of contracts. I develop a novel long-term contracting model where 1) the seller can price discriminate across buyers and time, and 2) the buyer can act opportunistically and simply take the goods and run whenever the delivery of the goods occurs before payment. Without access to external enforcement, the seller uses the value of the relationship itself to discipline the buyer’s behavior. I apply this modeling framework to study self-enforced relationships in the manufacturing supply chain in Ecuador, a middle-income country with slow commercial courts and concentrated sectors.

The paper has two novel empirical contributions. First, through the use of a structural econometric model, I provide the first evidence on the efficiency of long-term relationships in trade. The results show that relationships are highly inefficient early on. However, distortions vanish over time—highlighting the role of repeated informal agreements for value creation. Second, I counterfactually consider best-practice institutions (e.g., eliminating contracting frictions) and find that, surprisingly, in the medium and long-term they generate welfare losses relative to the second-best equilibrium. On the contrary, by addressing all the modeled frictions at once, efficiency increases.

I start by documenting six patterns that motivate the key ingredients in the model. First, the majority of trade is channeled through repeated relationships. Second, most transactions are financed by the vendor using trade-credit, even in new relationships. Third, relationships grow, both in terms of quantity and value, as they age. Fourth,
sellers offer significant quantity discounts—a 10% increase in quantity is associated with a 2% unit price decrease. Fifth, conditional on the quantity purchased, clients receive additional unit discounts as their relationship evolves—older buyers receiving up to 3% discounts relative to new ones. Given accounting markups of 20%, these discounts on quantities and age of relationships are economically significant. Finally, the survival probability of relationships increases in quantity and as relationships mature.

Standard models in the literature are not able to capture all of these patterns under realistic assumptions. For that reason, to account for these patterns and assess the efficiency of relationships over time, I develop a dynamic contracting model by embedding a non-linear pricing model with heterogeneous participation constraints (Jullien, 2000; Attanasio and Pastorino, 2020) into an infinitely repeated game with limited enforcement (Martimort et al., 2017; Pavoni et al., 2018; Marcet and Marimon, 2019). In the model, sellers and buyers with private heterogeneous demand meet randomly and have the opportunity to engage in repeated trade. The seller has all the bargaining power and proposes a dynamic contract of prices and quantities, for which they have commitment. Consistent with the data, the seller in the model finances all the transactions using trade-credit. Buyer heterogeneity provides incentives to price discriminate, so the seller offers menus of quantities and prices that satisfy *incentive compatibility* and induce revelation of the buyer asymmetric information.

Crucially, the buyer cannot commit to paying their debts and is subject to forward-looking *limited enforcement* constraints. The future stream of benefits created by the relationship must be large enough to secure the payment. To prevent a *take the goods and run* scenario, the seller must share a greater amount of surplus than otherwise. Thus, enforcement constraints could act against the seller’s profit maximizing incentives to distort trade downward through inefficiently low quantities. Matching the empirical picture described above, the optimal dynamic menu of quantities and prices in a setting with limited enforcement features *backloading*: both the total surplus generated by the relationship and the surplus captured by the buyer increase over time.

I employ a recursive Lagrangian approach (Pavoni et al., 2018; Marcet and Marimon, 2019) that allows me to characterize the optimal dynamic contract in terms of *past* and *present* limited enforcement Lagrange multipliers (LE multipliers). Current limited enforcement constraints are captured through present LE multipliers. Moreover, promises made about future levels of consumption to prevent default in the past are captured through past LE multipliers, which serve as promise-keeping constraints. In equilibrium, the optimal quantity allocations are then determined by a *modified virtual surplus*, which accounts for standard informational rents due to incentive compatibility as well as the shadow costs of binding enforcement constraints.

The paper specifies an econometric model directly from the theoretical model. It shows that the model’s parameters can be identified using cross-sectional information.
on the distribution of prices, quantities, age of the relationships, and a measure of the marginal cost of one seller. The dynamic identification results rely on the seller’s optimality conditions and the buyer’s dynamic first-order conditions for incentive compatibility (as in the static results of Luo et al., 2018 and Attanasio and Pastorino, 2020). In the model, the seller offers prices to induce the revelation of types and discriminate across different buyers, implying the observed price schedules at different quantities reveal information about the buyers’ heterogeneous types. Allocated quantities are determined through the seller’s first-order condition and are constructed so the gap between marginal prices and marginal costs respond to the seller’s current and past promises needed to satisfy enforcement constraints. Hence, conditional on past constraints and a measure for marginal costs, the variation in quantity and prices is informative about the shadow value of relaxing current enforcement constraints. Therefore, with observations on the cross-sectional paths of prices and quantities, it is possible to learn the distribution of unobserved buyer heterogeneity and the extent by which current quantities are distorted due to enforcement constraints.

I estimate the model using three administrative databases collected by the Ecuadorian government for tax purposes that match the objects in the theoretical model. I obtain pair-specific unit prices and quantities using a new electronic invoice database that contains all domestic sales for 49 manufacturing firms in the textile, pharmaceutical, and cement-product sectors for 2016-2017, each with a large number of buyers each year (median of 600). The age of relationships is inferred through the universe of firm-to-firm VAT database, which tracks the total volume of bilateral trade from 2008-2015. Lastly, a measure of seller’s costs comes from information on total variable costs (i.e., intermediate inputs expenditure and labor wages) contained in usual financial statements reported to the tax authority.

The model fits the data well, and the estimation reveals that enforcement concerns are relevant throughout the life-cycle of a relationship. Specifically, almost all new relationships have binding enforcement constraints. As relationships age, these constraints are relaxed, reflecting the increase in quantities coming from past promises made by the seller. Given the large number of trading partners, I explore the heterogeneity of enforcement constraints and find they differ significantly by buyers’ and sellers’ characteristics. For example, they are more likely to bind when the buyer is local rather than multinational or when the seller and buyer’s headquarters are far away.

I use the estimated parameters to assess the transactions’ efficiency at any point in time and learn about surplus division. I find that new relationships are, on average, at 30% of their first-best (i.e., frictionless) level. Efficiency increases over time, and those relationships lasting five years or longer are able to reach levels north of 80% efficiency. In terms of aggregate values, I find that sellers are greatly distorting quantities. When trading with new buyers, only 5% of suppliers procure aggregate levels that cannot be
distinguished from efficient output. Instead, the long-term aggregate output of 84% of sellers cannot be statistically distinguished from efficient levels. Regarding division of surplus, I find that sellers capture the majority of generated surplus (around 70%), although the largest buyers may capture up to 50% of the pie.

The paper then discusses counterfactual scenarios with counterintuitive implications. First, addressing seller market power or enforcement constraints alone, without addressing the other, leads to a lower total surplus, in the medium and longer-term. These results are direct manifestations of the theory of second-best (Lipsey and Lancaster, 1956). In the presence of one friction, the effect on welfare from eliminating one friction on its own is a priory ambiguous. In my context, each friction serves to counteract the other one. Second, by addressing both frictions at once, the results show that most relationships achieve higher total surplus and lower surplus for the seller.

This paper contributes to several strands of the theoretical and empirical literatures. First, I contribute to the theoretical and empirical literature on imperfect lending and contracting. The closest theoretical paper to mine is Martimort et al. (2017), which provides a theory of a two-sided limited enforcement problem in which buyers can default on debts and sellers can cheat on quality. In their setting, the buyer is the principal and increasingly shares a greater amount of surplus with the seller, implying dynamics where quantities and prices both increase. These dynamics do not match those observed in the setting I study, with frictions that are common in other parts of the developing world. In contrast, I consider a model where, besides the incentives to default, the buyer has private information about the value of the relationship and the seller has the bargaining power. Relative to the empirical literature, this is, to my knowledge, the first empirical paper to quantify the dynamic efficiency of self-enforced relationships.

This work also follows the theoretical and empirical literature related to price discrimination (Maskin and Riley, 1984; Jullien, 2000; Villas-Boas, 2004; Grennan, 2013; Luo et al., 2018; Attanasio and Pastorino, 2020; Marshall, 2020). The works by Luo et al. (2018) and Attanasio and Pastorino (2020) provide estimation methodology and identification results for static non-linear pricing problems, with and without binding participation constraints, respectively. This paper generalizes their models and estimation methods to a dynamic setting.

More generally, this paper relates to works in finance and development studying man...
ifestations of the theory of second-best (e.g., Petersen and Rajan, 1995; Genicot and Ray, 2006; Macchiavello and Morjaria, 2021). My work contributes to this strand of literature by suggesting that, empirically, fixing only one market friction may lead to welfare losses and showing that fixing both enforcement and seller market power could increase welfare. My counterfactual results also relate to the theoretical results of Genicot and Ray (2006), which shows that improving enforcement reduces the buyer’s expected payoff if the seller has the bargaining power, and of Troya-Martinez (2017), which finds total welfare decreases as enforcement quality increase beyond some intermediate level of enforcement.

Lastly, some of the empirical facts presented in Section 3 have been documented, individually, by previous works. The fact of relationship dynamics in quantities and prices has been previously documented for international trade by Heise (2019) and, partially, by Monarch and Schmidt-Eisenlohr (2017). The persistence of intra-national links is documented by Huneeus (2018) for Chile. Grennan (2013) and Marshall (2020) have documented price discrimination in the context of medical devices and wholesale food, respectively. Antras and Foley (2015), Garcia-Marin et al. (2019), and Amberg et al. (2020) have documented similar patterns of trade-credit issuance. To my knowledge, this paper is the first documenting relationship dynamics regarding prices and quantities intra-nationally, as well as the first documenting all of these facts in the same setting.

The remainder of the paper is organized as follows. Section 2 describes the context and presents summary statistics of the data. Section 3 offers the motivating facts that the model needs to match. Section 4 presents the model and its implications for dynamics. Section 5 discusses identification and the estimation procedure. Section 6 offers the estimated results, model fit, and discusses the performance of alternative models. Section 7 discusses welfare and three counterfactual exercises. Section 8 concludes.

2 Context, Interviews, and Data

Ecuador is an upper middle income country with weak enforcement of contracts and concentrated manufacturing markets. The World Bank Doing Business survey ranks Ecuador as a median country in terms of Contract Enforcement—capturing the courts’ efficiency in solving a quality dispute—and as one of the worst performers in terms of Insolvency measures—courts’ efficiency in solving a default in debts due to bankruptcy. Furthermore, its manufacturing sectors are highly concentrated, with average Herfindahl-Hirschman Indices of 0.6 for 6-digit economic codes, significantly higher than the concentration threshold of 0.25 used by the US Justice Department to flag highly concentrated markets.

3Macchiavello and Morjaria (2021) study the effects of increased competition in the coffee supply chain in Rwanda on welfare when trading partners engage in self-enforced agreements and find adverse effects of competition as it reduces parties ability to sustain the agreements. Similarly, Petersen and Rajan (1995) shows that increasing competition in bank lending when buyers have limited commitment to paying their debts actually hurts the buyers by decreasing overall volumes of lending.
2.1 Interviews
In order to gain insider knowledge of how manufacturing firms in Ecuador manage their relationships, I conducted hour-long free-form interviews with high-rank managers in 10 manufacturing firms in the Spring of 2019. The following points summarize the main takeaways:

- Relationships do not rely primarily on written contracts but rather on informal agreements. Although transactions are formally recorded when they occur, they tend to be managed without third-party enforcement; formal enforcement is costly and inefficient.\(^4\)
- Quality issues from suppliers were not highly relevant, as most inputs used tend to be very standardized.
- Enforcing payment of trade-credit transactions do require some investments, in terms of time and personnel, to pressure buyers to pay their debts.
- Most firms are aware that cash transactions obtain discounts (relative to trade-credit) and would like to take advantage of them, but often rely on trade-credit due to the lack of liquidity in the short-term.

This paper will not attempt to explain why these features exist but rather rely on them to understand the way they shape how on-going relationships are managed.

2.2 Administrative Data
The data used in this paper come from various administrative databases collected by Ecuador’s Servicio of Rentas Internas (IRS) for tax purposes.

2.2.1 VAT database
By law, since 2008, firms are required to report all of their firm-to-firm inputs and purchases with information on the identity of the buyer and seller through the B2B VAT system. I use the universe of business-to-business (B2B) VAT database for 2008-2015 to measure the lengths of relationships. In particular, I define age of relationship as the total number of years that the seller has sold some positive value to the buyer in the past. Given the first year of observation is 2008, age of relationship is censored at +9.

\(^4\)The Judicial Magazine of the Ecuadorian Government, available here, also provides evidence about the efficiency of the court system. I found two recent cases related to buyer default. (Case 1) Company attempts to collect 10K in debt from an invoice from March 2005. Company brings the case to court in October 2006. Final date of the case: October 2012. (Case 2) Company tries to collect 210K USD in debt from an invoice from January 2009. Company brings case to court in October 2011. Final date of the case: June 2015. In 2016, a new reform to the Código Orgánico General de Procesos was set in place to speed up debt collection. In theory, firms could bring cases to collect debts of up to 18K USD (in 2017) for a speedy audience. In practice, from interviews with the managers, this route was used as a last resort. The route is not exceptionally fast either. Personal estimates from 7K cases in the Civil Court in Quito, the capital, in 2017 show that it takes around 2 years to enforcement payment through the new expedited court system.
2.2.2 Electronic Invoicing

The primary data source for the analysis is the electronic invoicing (EI) system. In 2014, Ecuador started rolling out a new EI system to collect VAT information more consistently, requiring large firms to implement this new technology. By 2015, the largest 5000 firms were required to use the EI system for all sales. This system would send a copy of the transaction information to the buyer and government immediately after the transaction occurs. For each sale done by a firm in the system, the EI collects product-level information, including a bar-code identifier, product description, unit price, quantities, discounts, as well as transaction-level information, such as buyer unique national identifier and method of payment. Method of payment can be: cash, check, credit card, trade-credit offered by seller with trade-credit payment terms, amongst others.

My main estimating sample collects EI data for 49 manufacturing firms in textiles, pharmaceutical, and cement products for the years 2016-2017. The average firm in my sample is large (with 8,000 buyers), and has 24% of the market share in their 6-digit sector at the national level and 50% of the market share in their sector at their province level. The database’s coverage is good, with the average selling firm in my sample having more than 90% of the reported sales captured by the EI system. Interviews with managers in my sample indicate that most firms are using the invoices sent and received for internal accounting.

I classify a product as a bar-code identifier and description combination. I allocate any discount given in a transaction equally to all products purchased in the transaction by adjusting the product unit price by the discount. For instance, if discount offered amount to 5% of the transaction, I adjust reported unit prices of each product by 5%. Let $p_{ijgry}$ be the discount adjusted unit price and $q_{ijgry}$ be the reported quantity for buyer $i$ from seller $j$ for good $g$ in transaction $r$ during year $y$.

I define standardized unit prices at the transaction-product level $\tilde{p}_{ijgry}$ as

$$\tilde{p}_{ijgry} = \ln(p_{ijgry}) - \ln(p_{jgy}),$$

(1)

where $\ln(p_{jgy})$ is the average log discount adjusted price for the good $g$ of seller $j$ in year $y$. I define standardized quantity at the transaction-product level $\tilde{q}_{ijgry}$ in an analogous manner.

To obtain pair-year-level values of the standardized prices and quantities, I aggregate them by the respective share of total expenditures. Define $V_{ijy}$ as the total value of transactions between buyer $i$ and seller $j$ in year $y$. Let $s_{ijgry} = v_{ijgry}/V_{ijy}$ be the share of expenditure that good $g$ in transaction $r$ represents for the pair and $v_{ijgry} = p_{ijgry} * q_{ijgry}$ be the transfer value. Then, define pair-year level equivalents for the standardized prices and quantities as:

$$\tilde{p}_{ijy} = \sum_{r \in R_{ijy}} \sum_{g \in G_{ijy}} s_{ijgry} * \tilde{p}_{ijgry},$$

(2)
where $R_{ijy}$ is the set of all the transactions between $i$ and $j$ in year $y$ and $G_{ijry}$ is the set of all goods in transaction $r$.

These measures of standardized prices and quantities will be used in presenting motivating evidence of dynamics and patterns in prices and quantities. Their use indicate that product-specific differences across buyers do not drive the empirical facts.

Instead, for estimation, I will use the following definitions of prices and quantities, as they better match the structure of the model. For total quantity $q_{ijy}$, I sum over all reported quantities over all goods and all transactions:

$$q_{ijy} = \sum_{r \in R_{ijy}} \sum_{g \in G_{ijry}} q_{ijgy}.$$ (3)

For prices, I obtain average unit price by dividing total value of transactions by total quantity:

$$p_{ijy} = \frac{V_{ijy}}{q_{ijy}}.$$ (4)

This definition of prices lines up well with the weighted average of product-level discount inclusive prices, using expenditure weights, as shown in Online Appendix Figure OA1.

The total quantity produced by seller $j$ in year $y$ is given by $Q_{jy} = \sum_{i \in I_{jy}} q_{ijy}$, where $I_{jy}$ is the set of all buyers that transacted with the seller in the year.

Online Appendix Table OA2 presents summary statistics about quantities, values, and the number of buyers per seller obtained through this dataset.\footnote{Product-level variation of standardized prices and quantities is available in Supplemental Material, available on my website http://www.felipebrugues.com.}

### 2.2.3 Financial Statements

I complement this information with yearly data on expenditures and wage bill from financial statements for all sellers for 2016-2017.\footnote{In robustness exercises, I also use sales, exports, imports, total assets, total debt, total receivables, and total uncollectibles for all buyers and sellers in the data for 2008-2017. This data is obtained from the financial statements. I also add information on 6-digit sector code, GPS location of headquarters’ neighborhood, year founded, type of ownership (multinational, local, part of a business group), and whether the buyer and seller are vertically integrated. Online Appendix OA1 provides detailed description of the information captured in the databases. In summary, I find that sellers are larger, older, and have more direct contact with international trade than buyers. The median seller has higher markups (20 percent) than the median buyer (10 percent).} I use average variable cost $avc_{jy}$ for a firm $j$ in year $y$, defined as the sum of total expenditures and wages divided by total quantity $Q_{jy}$, as a proxy for marginal cost.

### 3 Motivating Evidence

In this section, I present evidence of how buyer-seller relationships operate in my setting. The data highlights three main ideas: i) trade depends heavily on past relationships and trade-credit, ii) as relationships age, quantities increase and prices decrease, and iii) at any
given point in time, larger purchases are met with lower prices. In Section 4, I propose a model that captures these dynamics by using a long-term contract, in which the seller can price discriminate across buyers and time, and where buyers can default on trade-credit debts with no legal repercussions.

**Fact 1: Large amount of trade occurs via repeated relationships**

Figure 1a shows that repeated relationships are important for the sellers in my sample. The blue bars show the average share of clients by length of relationship, whereas the green bars show the average share of total quantity sold. While around 35 percent of all buyer-seller pairs are with new buyers, only around 10 percent of trade is channeled through these new relationships. Instead, relationships that have been sustained for at least nine years represent less than 10 percent of all pairs but account for more than 30 percent of all trade.

**Fact 2: Most transactions occur via trade-credit**

The EI database contains information related to payment method, which specifies whether the transaction was financed by the seller and the terms of the credit in days. Here, I only consider whether any trade-credit was offered to the buyer, regardless of the terms of the agreement. Figure 1b plots the point estimate of the average, across sellers, of the share of relationships of a given age involving trade-credit at some point during a given year. Trade-credit usage is widespread, with around 85 percent of relationships receiving trade-credit in their first year of contact. By age 8, almost all relationships receive trade-credit during the year.

This fact has two important implications. First, the vendor is assuming a large share of the risks embedded in the transaction. In a weak legal enforcement framework, the buyer’s opportunistic action would likely imply all the direct costs of such action have to be directly absorbed by the seller. Second, the seller’s opportunistic actions, such as cheating in quality or quantity, are likely to be constrained (Smith, 1987). Post-delivery, the buyer can keep the value of the transaction as a guarantee of quality. For that reason, the terms of trade tilt in favor of the buyer when the seller finances transactions.

**Fact 3: Quantities increase as relationships age**

I now turn to provide evidence regarding the life-cycle of quantities in relationships in Figure 1c, which plots a binscatter regression of standardized log quantities $\tilde{q}_{ijgry}$ on dummies for the different ages of relationships in the cross-section. The figure shows that older relationships purchase more of a given product within a given year than younger relationships. In Online Appendix Figure OA2a, I verify that the dynamics are robust to using pair-fixed effects.\(^8\)

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7 Conditional on trade-credit being issued, the average maturity of the agreement is 29 days.

8 Note, however, that I only observe at most two years per pair. For that reason, within-pair growth uses partial information to reconstruct the whole path of quantities. To verify that the partial panel of
Figure 1: Motivating Facts

Notes: Subfigure a) presents the distribution of the average, across sellers, of the within-seller share of clients and quantity sold by age of relationship in 2016. Sub-figure b) plots the point estimate and 90% confidence interval of the average, across sellers, of the share of relationships of a given age involving trade-credit at some point during the year. Subfigure c) plots the cross-sectional evolution of standardized log quantities, with their corresponding 90% confidence intervals. Standardized log quantity is obtained by netting out the log average quantity in a given year for each seller-product. Standard errors are clustered at the seller-year. Subfigure d) shows the relationship between quantity purchased and standardized log unit price through binscatters of the measure of unit price against quantile of quantity by age of relationship. Standardized log unit prices are obtained by netting out the log average unit price in a given year for each seller-product. Quantiles of quantity are calculated for each seller-relationship age combination. Subfigure e) presents a binscatter of standardized log unit prices against years of relationship, controlling for a flexible spline of standardized log quantities. Standard errors are clustered at the seller-year level. Subfigure f) presents binscatters for the average survival rate of pairs at different ages and quantiles of quantity. Quantiles of quantities are created for each seller-age combination. Error bars are at the 90% level and reflect variation across sellers.

quantities is capturing correctly the growth of relationships, in Online Appendix Figure OA2d, I plot the path of total value transacted in relationships using both the partial panel captured in the EI database as well as a longer panel using VAT data for years 2008-2015. To correctly measure the age of a relationship in the VAT data, I drop relationships that start during the first year that a seller appeared in the data. Moreover, to correct for partial-year effects in exit (Bernard et al., 2017), I drop the last observation available for each pair. The figure shows that under both databases, the value transacted within pairs increases as they age. Moreover, the EI database’s partial panel accurately captures the full growth path observed in the VAT data.
**Fact 4: Quantity discounts for a given age of relationship**

Next, I study the relationship between prices and quantities. Given the differences in quantities sold by different manufacturers, I present quantities as quantiles, calculated within each seller and the following history types: i) new relationships, ii) relationships age 1-3, iii) relationships age 4+. Figure 1d presents a binscatterplot of the standardized unit price by quantiles of quantity. The standardization allows comparing quality-adjusted prices, as the variation is at the product-level. The figure shows that larger quantities receive lower quality-adjusted prices, regardless of the relationship’s age. This relationship is robust to looking at average unit prices and total quantities, as presented in Online Appendix Figure OA2b. To benchmark the size of quantity discounts, a 10% increase in total quantity purchased is associated with a 2% average price decrease (Online Appendix Table OA3).

**Fact 5: For a given quantity, older relationships pay lower unit prices**

Figure 1e shows the relationship between unit prices and relationship age through a binscatter regression of standardized log prices on age of relationship dummies, controlling for a flexible spline of standardized quantities. The figure reports that older relationships receive up to 3% more quality-adjusted additional discounts than new relationships. Given accounting markups of 20%, these discounts are economically significant as well.

These dynamic discounts over time are robust to controlling for pair fixed effects (Online Appendix Figure OA2c). Furthermore, in Online Appendix Table OA4 I replicate Figure 1e, and find that the relationship age effect is robust to additional controls to test for omitted variable bias. In particular, I control the buyer’s age, distance in kilometers between headquarters, size of the buyer (in sales, number of employees, assets), whether the buyer is a multinational, exporter, importer, or part of a business group. I also control for the importance of the relationship for the buyer (in terms of the supply share) or for the seller (in terms of demand share), in the spirit of Dhyne et al. (2022), to capture possible heterogeneous markups stemming from bilateral market power. Moreover, the effect is similar across the three types of industries considered (Online Appendix Table OA5).

In interpret this fact, together with the backloading of quantities, as evidence in favor of a model with limited enforcement of contracts. Such a model can accurately capture the price and quantity dynamics if the seller has a profit-maximizing incentive. By postponing the buyer’s share of the surplus, the seller can discipline the buyer’s behavior and maximize expected profits.

To facilitate this interpretation, rather than an efficiency gains story, I study two settings that relate to enforcement. First, in Online Appendix Table OA6, I explore dynamics by payment modality, both in the cross-section and panel. Intuitively, if incentives not to default drive the dynamics, then pay-in-advance buyers should not see any price
discounts. Effectively, in Column (1) and (3), I find that in both quality-adjusted price and average prices, pay-in-advance buyers see no discount over time. If anything, the estimates are positive, albeit statistically insignificant. Of course, for trade-credit buyers (Columns 2 and 3), discounts exist.

Second, in Online Appendix Table OA7, I explore dynamics by legal origin of multinational buyers. If the price dynamics were explained by efficiency gains, one should observe such pricing behavior for all multinationals, which have been shown to generate significant efficiency gains (Alfaro-Urena et al., 2022). While some multinationals observe price decreases over time, some other do not. Specifically, I find that multinationals from a common law origin, which tend to have better enforcement, do not obtain experience such dynamics. Given that common law origin is associated with better economic performance, common law multinational should be more likely to create productivity gains. Absent this effect in prices, it is unlikely that efficiency dynamics are driving the result.

**Fact 6: Relationships that trade more are more likely to survive**

Lastly, relationships are persistent. Figure 1f plots the share of relationships that survive from 2016 until 2017 by quantile of quantity in 2016 and age of relationship. The figure reports the share of new links that survive in red, in blue for links age 1-3, and in green for links age 4 or older. I find that around 40 percent of new relationships survive at least one more year, 60 percent of relationships age 1-3 survive, and more than 75 percent of relationships of 4 years or more survive. Moreover, within a relationship age, pairs that trade more volume are also more likely to survive from year to year.

## 4 Model of a Dynamic Contract

This section introduces the dynamic model. The model has three primary purposes: 1) allow for dispersion in quantity, 2) capture quantity discounts at any point in time, and 3) obtain the backloading of prices and quantities. I accomplish the first two goals by using heterogeneous private information on the buyer’s side. The model captures the backloading of prices and quantities by incorporating a limited enforcement constraint that prevents the buyer from defaulting on their trade-credit debts.

**Preliminaries**

Consider an infinitely repeated relationship between a seller (the principal) and a buyer (the agent). Time is indexed by $\tau \geq 0$ and we denote by $\delta < 1$ the common discount factor. Buyers’ preferences depend on a private information match attribute (or type) $\theta$, continuously distributed with support $[\underline{\theta}, \bar{\theta}]$, $\theta > 0$, cumulative distribution function $F(\theta)$ and probability density function $f(\theta)$. This match attribute is drawn at the beginning of the relationship and is kept constant over time. Although the parameter is private information, the distribution $F(\cdot)$ is common knowledge.
Relationships end due to exogenous shocks that happen at every period $\tau$ with probability $X(\theta)$. The exit probability $X(\cdot)$ is also common knowledge. Due to this, the type’s distribution evolves over time. Define $f_\tau(\theta) = f(\theta)(1 - X(\theta))^{\tau} / \int (f(m)(1 - X(m))^\tau)dm$ as the probability density function for time $\tau$ and $F_\tau(\theta)$ as its corresponding density function.

A trade profile stipulates an infinite array of transfers $t_\tau$ and quantities $q_\tau$ for each time period $\tau$, $\{t_\tau, q_\tau\}_{\tau=0}^{\infty}$. The trade profile gives the following discounted payoff to the principal

$$\sum_{\tau=0}^{\infty} \delta^{\tau}(t_\tau - cq_\tau)$$

and to the buyer

$$\sum_{\tau=0}^{\infty} \delta(\theta)^{\tau}(\theta v(q_\tau) - t_\tau),$$

where $v(\cdot)$ is the base return function and $\delta(\theta) \equiv \delta(1 - X(\theta))$. I consider $v(\cdot)$ strictly increasing and strictly concave.

### 4.1 Full Enforcement

As a benchmark, consider the case of full enforcement, both with symmetric and asymmetric information.

#### 4.1.1 Complete Information

Under complete information and full enforcement, the seller acts as a monopolist practicing first-degree price discrimination implementing a stationary contract $(t^{1d}(\theta), q^{1d}(\theta))$, which is defined as

$$\theta v'(q^{1d}(\theta)) = c \quad \text{and} \quad t^{1d}(\theta) = \theta v(q^{1d}(\theta)).$$

The seller offers first-best quantities but extracts all the rents from the buyer. This allocation is infinitely repeated over time.

#### 4.1.2 Asymmetric Information

The principal has commitment and wants to design a dynamic tariff scheme $t_\tau(\cdot)$ that maximizes their lifetime expected profit. The revelation principle applies to single-agent dynamic setups (Baron and Besanko, 1984; Sugaya and Wolitzky, Forthcoming), so there is no loss of generality in restricting the study to an infinite sequence menu $\{t(\theta), q(\theta)\}_{\theta}^{\infty}$ that induces the agent to report their true type.

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9 The model can accommodate for dynamic hazard rates $X_\tau(\theta)$.

10 Throughout the next sections, I will use the terms transfers and tariffs interchangeably.

11 This property of the buyer’s return function can be micro-founded by using diminishing returns in production for one input, keeping at least one other input fixed. This assumption is common in the literature. For instance, standard production function estimation generally assumes that capital is set one year in advance (e.g., Levinsohn and Petrin, 2003).
The theoretical insights from Baron and Besanko (1984) apply in this setup. The optimal dynamic contract with full enforcement is equal to repeated Baron-Myerson static contracts with quantities determined by:

\[ \theta v'(q^e_\tau) = c - \frac{1 - F_\tau(\theta)}{f_\tau(\theta)} v(q^e_\tau(\theta)), \]  

(PE)

and tariffs such that

\[ t^e_\tau(\theta) = \theta v(q^e_\tau(\theta)) - \int_\theta^\theta v(q^e_\tau(x)) dx. \]

It is possible to show that under positive selection (i.e., \( X'(\theta) < 0, \forall \theta \)), average and type-specific quantities decrease over time. Similarly, average and type-specific unit prices can be shown to increase. Instead, without selection patterns (i.e., \( X'(\theta) = 0, \forall \theta \)), the optimal full enforcement contract with asymmetric information is stationary.

### 4.2 Limited Enforcement

While the seller can commit fully to the long-term contract, the buyer can act opportunistically. I assume that, in each period, the seller first delivers the goods and has to wait for the buyer to transfer the promised amount before the end of the period, effectively offering trade-credit to the buyer in every transaction. While this assumption is strong, it reduces the complexity of the problem and data shown in Section 3 shows trade-credit is extremely common, if not the norm.

The direct mechanism \( C(\theta) = \{q_\tau(\theta), t_\tau(\theta)\}_{\tau=0}^\infty \) stipulates quantities and post-delivery transfers in each period for agent reporting type \( \theta \). The seller offers the menu of \( \{\theta, C(\theta)\}_{\theta \in \Theta} \), with combinations of available reporting types and corresponding allocations.

#### 4.2.1 Timing

The contracting game takes places in the following order:

1. Prior to trade, at \( \tau = 0 \), the buyer observes their private type \( \theta \). The seller offers the mechanisms menu \( \{C(\theta)\} \). The buyer either accepts or rejects the offer. If they accept, they report type \( \hat{\theta} \). If they reject, both the seller and buyer receive their outside options, normalized to 0.

2. In each trading period \( \tau \geq 0 \):

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12 Theorem 4' offers the results for fully persistent types in an infinite horizon model.
13 With positive selection, informational rents given to middle-types decrease, as the distribution is shifting towards higher-types \( F_\tau(\theta) > F_{\tau+1}(\theta) \). In order to incentivize the highest types still active, middle-types will be distorted downwards in the future. Marginal unit prices are given by \( p(q(\theta)) = c + \frac{(1 - F_\tau(\theta))}{f_\tau(\theta)} \) (Armstrong, 2016), which will be generally larger for each \( \theta \), and as such, average price will be larger at each \( q \).
14 For the buyer, this normalization is not restrictive if they use a production function that mixes the inputs linearly or if the supplier is a true monopolist. For the seller, the normalization is not restrictive if there are no scale economies.
• The seller produces and delivers $q_{r}(\hat{\theta})$.
• The post-delivery payment $t_{r}(\hat{\theta})$ is paid by the buyer, or they breach the contract.
• Following a breach on the buyer’s side, the contract is terminated.

As it will made clear below, the contract considered is default-free, through the use of enforcement constraints, and features no renegotiation. Since default never occurs in equilibrium, there is no loss in assuming that the seller terminates trade following a breach (Abreu, 1988; Levin, 2003).

4.2.2 Constraints

Let us now characterize the set of constraints in the main problem. The set of constraints contain the usual individual rationality and incentive compatibility constraints of adverse selection problems. This setting’s novelty is to include an additional enforcement constraint in each trading period, which acts as an endogenously determined participation constraint. Each of the enforcement constraints will ensure the buyer will not default in the specific time period.

Buyer’s Incentive Compatibility

Under the assumption of perfectly persistent types, as in Martimort et al. (2017), incentive compatibility requires that the agent evaluates their lifetime return:

$$\sum_{\tau=0}^{\infty} \delta(\theta)^{\tau} u_{r}(\theta) \geq \sum_{\tau=0}^{\infty} \delta(\theta)^{\tau} [\theta v(q_{r}(\hat{\theta})) - t_{r}(\hat{\theta})] \quad \forall \theta, \hat{\theta},$$

where $u_{r}(\theta) = \theta v(q_{r}(\theta)) - t_{r}(\theta)$.

Buyer’s Limited Enforcement Constraint

The key friction in the model is the limited enforcement of the trade-credit contracts, which allows for the possibility of buyer’s default. Under the assumption of contracting termination following a breach, a default-free menu satisfies the limited enforcement constraint of the buyer:

$$t_{r}(\theta) \leq \sum_{s=1}^{\infty} \delta(\theta)^{s} u_{s}(\theta) \quad \forall \theta, \tau.$$  

The condition requires that the costs of breaking the relationship, in terms of the forgone opportunities of trade, have to be greater than the benefits from breaching the contract.

The buyer’s LE-B constraint at $\tau = 0$ implies the individual rationality constraint required for buyer participation in trade.\textsuperscript{15} For that reason, only LE-B and IC-B are considered. From this, it follows that ex-ante trade under limited enforcement should

\textsuperscript{15}A mechanism $C$ is individually rational if the participation constraint at $\tau = 0$ holds: $\sum_{\tau=0}^{\infty} \delta(\theta)^{\tau} u_{r}(\theta)) \geq 0 \quad \forall \theta$. To see how LE-B implies this, add $u_{0}(\theta)$ on both sides and note that $u(\theta) + t_{r}(\theta) = \theta v(q_{r}(\theta)) > 0.$
leave participating buyers weakly better than under perfect enforcement whenever the
seller has the bargaining power.

4.2.3 Optimal Contract with Limited Enforcement

Denote total surplus as $s(\theta, q) = \theta v(q) - cq$. The principal’s problem becomes

$$\max_{\{u_{\tau}(\theta), q_{\tau}(0)\}} \sum_{\tau=0}^{\infty} \delta^{\tau} \int_{\theta}^{\theta} [s(\theta, q_{\tau}(\theta)) - u_{\tau}(\theta)] f_{\tau}(\theta) d\theta,$$

such that IC-B and LE-B are satisfied. That is, the objective of the seller is to maximize
total surplus while reducing the share of surplus given to the seller as much as possible
without violating the constraints.

The solution in a static setting (e.g., in Jullien (2000)) follows the first-order approach
of Mirrlees (1971), which substitutes the global incentive compatibility constraint with a
local one. Recent results in dynamic mechanism design (Pavan et al., 2014; Battaglini
and Lamba, 2019) show that a dynamic envelope theorem for the relaxed problem can be
used to characterize under certain conditions the global solution to the full contract. In
particular, Battaglini and Lamba (2019) argue that if types are fully persistent, strictly
monotonic contracts (i.e., those with $q'_{\tau}(\theta) > 0$ for all $\theta$ and $\tau$) will be globally incentive
compatible. Throughout this section, I will assume that allocated quantities satisfy this
monotonicity property.

Following Pavan et al. (2014), an implementable menu satisfies dynamic incentive-
compatibility if it satisfies the dynamic envelope formula:

$$\sum_{\tau=0}^{\infty} \delta(\theta)^{\tau} u'_{\tau}(\theta) = \sum_{\tau=0}^{\infty} \delta(\theta)^{\tau} v(q_{\tau}(\theta)),$$

for any arbitrary $0 < \delta(\theta) < 1$ function and $u'_{\tau}(\theta) \equiv du_{\tau}(\theta)/d\theta$. Substituting the envelope
condition 7 with $\delta(\theta) = \delta$ into the seller’s problem SP yields

$$\sum_{\tau=0}^{\infty} \delta^{\tau} \int_{0}^{\theta} [s(\theta, q_{\tau}(x)) - \int_{0}^{\theta} v(q_{\tau}(x))dx] f_{\tau}(\theta) d\theta - \sum_{\tau=0}^{\infty} \delta^{\tau} u_{\tau}(\theta).$$

The return term of the buyer acknowledges the rents that have to be given to higher types
in order to preserve incentive compatibility.

I follow Jullien (2000) and write the problem in Lagrangian-type form. For this formulation,
the dynamic LE-B constraint at time $\tau$ is given by:

$$\int_{0}^{\theta} \left\{ \sum_{s=1}^{\infty} \delta^{s}(1 - X(\theta))^s u_{\tau+s}(\theta) - [\theta v(q_{\tau}(\theta)) - u_{\tau}(\theta)] \right\} d\Gamma_{\tau}(\theta) = 0, \quad \text{(Lagrangian-D-LE)}$$

where $\Gamma_{\tau}(\theta) = \int_{0}^{\theta} \gamma_{\tau}(x) dx$ is the cumulative LE multiplier with derivative $\gamma_{\tau}(\theta)$. The
derivative $\gamma_{\tau}(\theta) > 0$ whenever the limited enforcement constraint binds and it captures
the shadow value of the enforcement constraint at $\theta$. The cumulative multiplier $\Gamma_{\tau}(\theta)$
captures the extent by which trade is distorted by limited enforcement. It represents the
shadow value of relaxing the enforcement constraints uniformly from $\theta$ to $\theta$, capturing the amount of profits lost by the seller due to enforcement incentives. As extending $\theta$ increases the set on which the enforcement constrained is relaxed, $\Gamma_\tau$ is nonnegative and nondecreasing. Moreover, by relaxing uniformly the constraints, the seller can reduce the buyers’ net returns by keeping quantities unchanged, so $\Gamma_\tau(\bar{\theta}) = 1$.

Thus, the cumulative multiplier has the properties of a cumulative distribution function.

After manipulating the limited enforcement constraints, one can obtain the full Lagrangian maximand:

$$\sum_{\tau=0}^{\infty} \delta^\tau \int_0^\theta \left[ s_\tau(\theta, q_\tau(\theta)) - v(q_\tau(\theta)) \right] \Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1} (1 - \Gamma_s(\theta))(\tilde{\Gamma}_s(\bar{\theta}) + \theta \gamma_\tau(\theta)) f_\tau(\theta) d\theta,$$

with the corresponding slackness condition Lagrangian-D-LE and where $\Gamma_s^\tau(\theta)$ is the conditional cumulative LE multiplier constraint defined by

$$\Gamma_s^\tau(\theta) = \int_\theta^\theta (1 - X(x))^{\tau-s} \gamma_s(x) dx / \tilde{\Gamma}_s^\tau(\bar{\theta}),$$

for $\tilde{\Gamma}_s^\tau(\bar{\theta}) = \int (1 - X(\theta))^{\tau-s} \gamma_s(\theta) d\theta$. The conditional cumulative multiplier constraint adjusts for the likelihood that a given $\theta$ has survived $\tau - s$ periods, assigning lower weights to $\theta$’s that are less likely to survive.

The corresponding seller’s first order condition determining the allocation rule at any relationship tenure $\tau$ is:

$$\theta v'(q_\tau(\theta)) - c = \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1} (1 - \Gamma_s^\tau(\theta)) \tilde{\Gamma}_s^\tau(\bar{\theta}) + \theta \gamma_\tau(\theta)}{f_\tau(\theta)} v'(q_\tau(\theta)). \quad \text{(SFOC)}$$

The allocation equation responds to intuitive forces. For expositional purposes, assume that the breakup probability is zero for all types, $X(\theta) = 0$ for all $\theta$. Then, $\Gamma_\tau^\tau(\theta) = \Gamma_s^\tau(\theta)$, $\tilde{\Gamma}_s^\tau(\bar{\theta}) = 1$, $F_\tau(\theta) = F(\theta)$, $f_\tau(\theta) = f(\theta)$. Assume as well that $v(q) = kq^\beta$. The equation can be written as:

$$q_\tau(\theta)^{1-\beta} = \frac{k\beta}{c} \left[ \theta - \frac{1 - F(\theta)}{f(\theta)} \right] + \frac{\theta \gamma_\tau(\theta)}{f(\theta)} + \frac{1 - \Gamma_\tau(\theta)}{f(\theta)} + \frac{\sum_{s=0}^{\tau-1} (1 - \Gamma_s(\theta)) \tilde{\Gamma}_s(\bar{\theta})}{f(\theta)} \right] \quad \text{(Q-CES)}$$

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16 In Online Appendix Section OA5, I show formally that $\Gamma_\tau(\bar{\theta}) = 1$.

17 Pre-multiply each constraint by $\delta^\tau$ and sum over $\tau$. Reorder internal summations, substitute in the dynamic envelope condition, and eliminate constant terms to obtain:

$$\sum_{\tau=0}^{\infty} \delta^\tau \int_0^\theta \int_\theta^\theta v(q_\tau(\theta)) dx \sum_{s=0}^{\tau-1} (1 - X(\theta))^{\tau-s} \tilde{\Gamma}_s^\tau(\bar{\theta})$$

$$- \sum_{\tau=0}^{\infty} \delta^\tau \int_\theta^\theta [\theta v(q_\tau(\theta)) - \int_\theta^\theta v(q_\tau(\theta)) dx] d\Gamma_\tau(\theta). \quad \text{(9)}$$

Then integrate by parts.
which resembles the usual solution to an adverse selection problem in which the allocation is determined by an inverse markup ($\mu$) rule adjusted by the modified virtual surplus, which accounts for necessary rents due to incentive compatibility and due to the limited enforcement constraint.

In particular, the incentive compatibility constraint forces the seller to give higher quantities to higher types through $F(\theta)$ as informational rents. Moreover, when the current limited enforcement constraint is binding ($\gamma(\theta) > 0$), it limits the volume of trade. Keeping the future stream of quantities constant, if the buyer is on the verge of defaulting, the seller needs to reduce tariffs now. But, given profit maximizing incentives, the seller must also decrease quantities. Hence, enforcement concerns decrease contemporaneous quantities. Yet, at the same time, a countervailing force exists: to preserve incentive compatibility and prevent low-types from pretending to be higher-types, quantities are uniformly shifted upwards by $1 - \Gamma(\theta)$.

Importantly, relative to Jullien (2000), the critical distinction here is the addition of past cumulative multipliers ($\sum_{t=0}^{T-1}(1 - \Gamma_s(\theta))$), which generate the backloading of quantities. This multiplier serves a promise-keeping constraint, where types for which their limited enforcement constraint was binding in the past, receive higher quantities in the present. A similar result is offered in Martimort et al. (2017) for a discrete number of types. With exogenous exit ($X > 0$), promises made in the distant past weigh less now. However, if relationships never end, promises made in the past shift trade levels forever, as in Marcet and Marimon (1992).

The equilibrium combination of $\Gamma_\tau(\theta)$, $\Gamma_s(\theta)$, and $\theta\gamma(\theta)$ determine whether quantity allocated is greater or lower than under full enforcement. Furthermore, as usual, allocated quantities decrease in the markup that a seller would charge under linear monopolist pricing.

The results of Pavan et al. (2014) allow us to construct the transfers $t_\tau(\theta)$ satisfying the necessary first-order conditions with the corresponding allocation rule specified in SFOC. Specifically, if the contract allocation satisfies a strict monotonicity assumption, the following transfer rule satisfies the buyer’s dynamic envelope formula:

$$t_\tau(\theta) = \theta v(q_\tau(\theta)) - \int_\theta^\theta v(q(x))dx - u_\tau(\theta),$$

(t-RULE)

and so the derivative of the transfer rule with respect to type is

$$t'_\tau(\theta) = \theta v'(q_\tau(\theta))q'_\tau(\theta).$$

(t-RULEp)

4.2.4 Non-Stationary Equilibrium

This paper does not attempt to characterize the full optimal dynamic contract but instead conjecture about its existence and use data to learn about the primitives of the

\[\text{Pavan et al. (2014) use a weaker assumption, integral monotonicity, which is implied by the strict monotonicity assumption.}\]
model. Yet, I prove the optimal contract cannot be stationary in Online Appendix OA4 in two steps. First, I prove the existence of a unique stationary equilibrium. Second, I show that a non-stationary deviation exists that dominates the stationary equilibrium. For that reason, if an optimal contract exists, it must be non-stationary.

4.2.5 Model Dynamics and Static Efficiency

In Online Appendix Section OA6, I further discuss model dynamics and efficiency. In particular, I provide sufficient conditions to observe backloading of quantities and prices, as well as cross-sectional quantity discounts. Furthermore, I show that the model converges to a stationary long-term equilibrium, which may be efficient or inefficient, both in terms of under-consumption or over-consumption. That is, the model does not imply long-term efficiency by construction. I then compare the efficiency of the limited enforcement contract to that under perfect enforcement, and find that, despite its theoretical ambiguity, long-term relational contracts may potentially generate greater levels of efficiency than allocations under perfect enforcement.19

5 Identification and Estimation of Dynamic Contracts

This section discusses identification of the model primitives $\theta, v(\cdot)$ and $\Gamma_\tau(\cdot)$. Moreover, it provides a detailed discussion regarding the identification assumptions and provides guidance on how model mispecification would affect the results. Lastly, it describes an estimation procedure that directly relies on the main identification equations to recover the model primitives from the available data.

5.1 Identification

For each seller in a given year, the observables are unit prices $p_\tau(q)$ (or transfers $t_\tau(q)$) and quantities $q_\tau$ for different buyers with relationship age $\tau$, as well as marginal costs $c$. Throughout this section, I abstract away from the possibility of exogenous breakups. The possibility of breakups will be reintroduced in estimation.20

As shown in Section 4, the dynamic contract is a complex object. Rather than deriving the full equilibrium contract by forward-iteration, I rely on the following assumption for identification.

Identification Assumption 1. Each seller offers a unique menu of dynamic contracts to all buyers, and such menu satisfies equations SFOC and $t$-RULEp for all $\theta$ and $\tau$.

19To see a solved example, please refer to Supplemental Material Section SM4, which presents a solved two-type example illustrating these features.

20As exogenous breakups can be directly estimated in the data, they would be treated as known during identification. Their inclusion would only add complexity in the notation without adding substantial insights regarding identification.
Under this assumption, I can collapse all information about future unobserved quantities and transfers into the limited enforcement multipliers. The seller is aware of the solution and the future promises and I exploit this knowledge to learn about enforcement distortions. Although the assumption is strong, it is often used in the identification of dynamic games, as these types of games may have multiple equilibria (Aguirregabiria and Nevo, 2013).

For identification, I exploit the fact that the mapping from agent type $\theta$ to quantity $q_\tau$ is strictly monotone and write the first-order condition of the seller SFOC and the derivative of the transfer rule of the buyer $t$-RULE in terms of quantiles (Luo et al., 2018; Luo, 2018):

$$\theta_\tau(\alpha)q'(q_\tau(\alpha)) - c =$$

$$\left[\Gamma_\tau(\alpha) - \alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha)) + \frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)}\gamma_\tau(\alpha)\right]\theta_\tau(\alpha)q'(q_\tau(\alpha))\frac{\theta'_\tau(\alpha)}{\theta(\alpha)},$$

where $\alpha \in [0, 1]$ and I used the fact that observed price schedule can be mapped to the model transfer schedule by $T_\tau(q_\tau(\theta(\alpha))) = t_\tau(\theta(\alpha))$. Moreover, $\theta_\tau(\alpha)$ and $q_\tau(\alpha)$ are the $\alpha$-quantiles of the agent’s type and quantity at tenure $\tau$, respectively. Notice as well that I have used $f_\tau(\theta(\alpha)) = 1/\theta'_\tau(\alpha)$ and $\gamma_\tau(\theta(\alpha)) = \gamma_\tau(\alpha)/\theta'_\tau(\alpha)$.

Together, the key identification equation becomes:

$$\frac{T'_\tau(q_\tau(\alpha)) - c}{T'_\tau(q_\tau(\alpha))} = \frac{\theta'_\tau(\alpha)}{\theta_\tau(\alpha)}\left[\Gamma_\tau(\alpha) - \alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha))\right] + \gamma_\tau(\alpha),$$

where $\theta_\tau(\cdot)$, $\theta'_\tau(\cdot)$, $\Gamma_\tau(\cdot)$, and $\gamma_\tau(\cdot)$ are unknown. The price schedule $T_\tau(\cdot)$ and its derivatives are nonparametrically identified from information on prices and quantities alone, so in this section, I treat them as known. Moreover, I treat $c$ as known.

**Identification of the Limited Enforcement Multiplier $\Gamma_\tau(\theta)$**

The identification argument is recursive and takes the the primitives at time $s < \tau$, and in particular, $\Gamma_s(\alpha)$, as known.

Define $\Xi(\alpha) = \Gamma_\tau(\alpha) + \theta_\tau(\alpha)/\theta'_\tau(\alpha)\gamma_\tau(\alpha)$. Substituting in and reordering, equation 12 becomes:

$$\Xi(\alpha) = \alpha + \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha)) + \frac{T'_\tau(q_\tau(\alpha)) - c \theta_\tau(\alpha)}{T'_\tau(q_\tau(\alpha))} \theta'_\tau(\alpha).$$

As $\theta_\tau(\alpha) > 0$ and $\theta'_\tau(\alpha) > 0$, $\Xi(\alpha)$ is set identified. In particular,

$$\Xi(\alpha) \in \begin{cases} 
[0, \alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha))] & \text{if } T'_\tau(q_\tau(\alpha)) < c, \\
[\alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha)), 1] & \text{if } T'_\tau(q_\tau(\alpha)) \geq c, 
\end{cases}$$

where $\Gamma_s(\alpha)$ is assumed to be known.
For every value $\Xi_\tau(\alpha)$, there is a unique value for $\theta_\tau(\alpha)/\theta'_\tau(\alpha)$. Therefore, for each combination of $\{\Xi_\tau(\alpha)\}_{\alpha\in[0,1]}$, $\Gamma_\tau(\alpha)$ is identified from the solution of the differential equation:

$$\gamma_\tau(\alpha) + \Gamma_\tau(\alpha) \frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)} = \Xi_\tau(\alpha) \frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)}, \quad (14)$$

after defining a boundary condition for $\Gamma_\tau(\alpha)$, which is shown in Online Appendix OA5 to be $\Gamma_\tau(1) = 1$. For that reason, $\Gamma_\tau(\cdot)$ is set identified.\(^{21}\)

However, by making the parametric assumption on the return function $v(q) = kq^\beta$ for $k > 0$ and $\beta \in (0, 1)$, the multipliers $\Gamma_\tau(\cdot)$ are point identified.\(^{22}\) Appendix A provides the details. Generally speaking, $\Gamma_\tau(\cdot)$ is point identified up to a function $A(q) = -v''(q)/v'(q)$. By parametrizing $v(q) = kq^\beta$, the function $A(q) = (1 - \beta)/q$ depends only on one parameter, which is identified from observations of prices, quantities, and marginal cost for the lowest and highest consumption buyers. Hence, through the assumed parametrization, $\Gamma_\tau(\cdot)$ is point identified from observations of prices, quantities, and marginal cost.

For estimation, I follow the approach of Attanasio and Pastorino (2020), which consider a parametrization of $\Gamma_\tau(\cdot)$ as a flexible function of $q_\tau$ rather than parametrizing $v(\cdot)$. Throughout the remainder of the section, I consider $\Gamma_\tau(\cdot)$ as point identified.

**Identification of Types $\theta$**

Using the allocation equation at $\tau$ and the fact that $\delta \ln(\theta_\tau(\alpha))/\delta \alpha = \theta'_\tau(\alpha)/\theta_\tau(\alpha)$, I obtain the following expression for $\theta_\tau(\alpha)$:

$$\ln(\theta_\tau(\alpha)) = \ln(\theta_\tau) + \int_0^\alpha \frac{\delta \ln(\theta_\tau(x))}{\delta x} dx \quad (15)$$

$$= \ln(\theta_\tau) + \int_0^\alpha \frac{1}{\Gamma_\tau(x) - x - \sum_{s=0}^{\tau-1} (1 - \Gamma_s(x))} \left[ \frac{T'_\tau(q_\tau(x))}{T'_\tau(q_\tau(x))} - \gamma_\tau(x) \right] dx, \quad (16)$$

\(^{21}\) If $\Gamma_s(\alpha)$ is taken to be a set, then the identification set for $\Xi_\tau(\alpha)$ should be defined as:

$$\Xi_\tau(\alpha) \in \left\{ 0, \alpha - \sum_{s=0}^{\tau-1} (1 - \Gamma_s^{SUP}(\alpha)) \right\} \text{ if } T'_\tau(q_\tau(\alpha)) < c,$$

$$\left\{ \alpha - \sum_{s=0}^{\tau-1} (1 - \Gamma_s^{INF}(\alpha)), 1 \right\} \text{ if } T'_\tau(q_\tau(\alpha)) \geq c,$$

where $\Gamma_s^{SUP}(\alpha)$ is the supremum and $\Gamma_s^{INF}(\alpha)$ is the infimum in identified set for $\Gamma_s(\alpha)$. Although the bounds for $\Xi_\tau(\alpha)$ are wider, the identification argument for $\Gamma_\tau(\cdot)$ remains unchanged. For every value $\Xi_\tau(\alpha)$ and $\sum_{s=0}^{\tau-1} (1 - \Gamma_s(\alpha))$, there is a unique value for $\theta_\tau(\alpha)/\theta'_\tau(\alpha)$. Therefore, for each combination of $\{\Xi_\tau(\alpha), \sum_{s=0}^{\tau-1} (1 - \Gamma_s(\alpha))\}_{\alpha\in[0,1]}$, $\Gamma_\tau(\theta)$ is identified from the solution of the differential equation 14.

\(^{22}\) Luo (2018) studies the nonparametric identification of this model and of $\Gamma_\tau(\alpha)$ in particular with observations on prices and quantities alone. They find that this model is nonparametrically identified if one can find an alternative efficient market, for which $\Gamma_\tau(\alpha) = 1$ for all $\alpha$, in order to learn about $\theta'_\tau(\alpha)/\theta_\tau(\alpha)$. With information on $\theta'_\tau(\alpha)/\theta_\tau(\alpha)$ in hand, $\Gamma_\tau(\alpha)$ is nonparametrically identified from information on transfers and prices alone. This approach is not feasible in my setting as each seller is considered a market, and it is impossible to find something that could be regarded as an alternative efficient market for each seller.

21
which identifies the quantile function of type $\theta_r(\cdot)$ up to $\theta_r$ and $\Gamma_r(\alpha)$. Making the scale normalization on types $\theta_r \equiv \theta = 1$, the quantile function for types becomes:

$$
\theta_r(\alpha) = \exp \left( \int_0^\alpha \frac{1}{\Gamma_r(x)} x - \sum_{s=0}^{r-1} (1 - \Gamma_s(x)) \left[ \frac{T_r'(q_r(x)) - c}{T_r'(q_r(x))} - \gamma_r(x) \right] dx \right). \quad (17)
$$

As Luo et al. (2018) show, $\theta = 1$ is a normalization for a general function $v(\cdot)$. Under a parametrization $v(q) = kq^\beta$, which provides point identification for $\Gamma_r(\cdot)$, $\theta = 1$ is also a normalization as it suffices to multiply $k$ by the normalization constant to obtain an observationally equivalent structure.

The distribution $f_r(\theta)$ is identified from $\theta_r'(\alpha)$ since $f_r(\theta) = 1/\theta_r'(\alpha)$ and $\theta_r'(\alpha)$ is obtained from

$$
\theta_r'(\alpha) = \frac{\theta_r(\alpha)}{\Gamma_r(\alpha) - \alpha - \sum_{s=0}^{r-1} (1 - \Gamma_s(\alpha)) \left[ \frac{T_r'(q_r(\alpha)) - c}{T_r'(q_r(\alpha))} - \gamma_r(\alpha) \right]}.
$$

(18)

**Identification of the Base Return Function $v(\cdot)$**

The base return function $v(q_r(\alpha))$ is identified in two steps under the assumed parametrization. First, the elasticity $\beta$ is identified from observations of quantities, prices, and marginal costs, as detailed in Appendix A. The level shifter $k$ is identified using the derivative of the transfer rule $T_r'(q_r(\alpha)) = \theta_r(\alpha) v'(q_r(\alpha)) = \theta_r(\alpha) k \beta q_r(\alpha)^{\beta-1}$, as $\theta_r(\alpha)$ and $\beta$ are identified, while $q_r(\alpha)$ and $T_r'(q_r(\alpha))$ are known.

**Identification of the Future Value of the Relationship**

For types with binding enforcement constraints, $\gamma_r(\theta(\alpha))) > 0$, the future value of the relationship\(^{23}\) is identified from the observed transfers and the binding enforcement constraint:

$$
T_r(q_r(\alpha)) = \delta \sum_{s=1}^\infty (\theta v(q_{r+s}(\alpha)) - T_{r+s}(q_{r+s}(\alpha))).
$$

5.2 Discussion

My methodology relies on the fact that the seller knows the solution to the optimal contract and that such contract must satisfy the first-order conditions of the seller and the buyer. Observable distortions in quantity relative to an unconstrained world are then informative about informational and enforcement rents. This approach is beneficial as I can identify and estimate the model without solving it. A full-solution approach would require to forward-iterate the model repeatedly for each specific type, generating computing burden and limiting the dimension of the type-space.

This assumption is also useful in case of model mis specification. Specifically, it allows the buyer to be subject to outside options, which the econometrician does not observe, as long as the seller is aware of them. These outside options will be captured in the

\(^{23}\)Similar to one of the objectives of Macchiavello and Morjaria (2015).
enforcement constraints. The econometrician will not be able to separate the effects of outside options from those from future promises. Therefore, under model mispecification, the econometrician will not be able to identify the future value of the relationship for types with binding constraints. However, this does not create identification issues for the primitives relevant for welfare analysis.\textsuperscript{24}

One important disadvantage of this approach is the type of counterfactuals that can be considered. In particular, it rules out counterfactuals that consider dynamic quantities. After all, obtaining counterfactual dynamic quantities would require to solve the model iterative. Nonetheless, as the focus of this paper is to provide evidence on the efficiency of \textit{actual} trade, the methodology provides important insights for this question.\textsuperscript{25}

The methodology also relies on the commitment of the seller to the mechanism. Starting in the second period, the seller knows the asymmetric information of the buyer, but does not change the mechanism to improve their profits by reducing the informational rents given to the buyers. This assumption is arguably strong in a setting where one of the parties can defect.

Despite this disadvantage, I consider the commitment solution as relevant for three reasons. First, the commitment solution is a natural benchmark. Indeed, it is often used in model of sovereign default (Dovis, 2019), firm dynamics (Roldan-Blanco and Gilbukh, 2021), and risk-sharing in developing countries (Ábrahám and Laczó, 2018). The new identification results for dynamic contracts developed here thus have a broad range of potential applications. Second, methodologically, solving dynamic mechanism problems with limited commitment on the principal’s side is complex. Only recently, Doval and Skreta (2020) developed a new toolkit to address this issue. I have explored using this toolkit. However, by allowing the principal to have limited commitment, the new solution will require the introduction of new sequential rationality constraints. This would entangle the enforcement and commitment problem, generating identification issues in both constraints. Lastly, the concerns of the commitment problem can be alleviated by obtaining longer panel data that tracks the evolution of quantities for each pair over time. With such data, one can relax the assumption of fully persistent types and use a more flexible Markov approach. If that is the case, the seller will never be able to fully learn the buyer type and the interest in updating the allocation rule will be reduced.

\textsuperscript{24}Model mispecification will affect the \textit{model} solution for the equilibrium transfers. For instance, if buyers are subject to a constant outside option, equilibrium transfers will be lower, by a size equal to the outside option. However, marginal prices are not affected, and therefore, the primitives identified from marginal prices—the base marginal return jointly with the type—are not affected either.

\textsuperscript{25}Model mispecification in terms of outside options also affects the counterfactuals of different enforcement or pricing regimes. If the outside options are constant, the counterfactuals will be correct in terms of efficiency. Surplus division will be biased towards the seller. If outside options are heterogeneous, the counterfactual efficiency will also be affected. Yet, the direction of the bias is uncertain ex-ante, as it depends on the distribution of types and the curvature of the return function.
5.3 Estimation

The main estimation steps are based on identification equations 12 and 17. I estimate the equations separately for each seller $j$ and tenure $\tau$ using $N_{j\tau}$ observations for each year of observation available.

The general structure of the estimation procedure is as follows. First, I perform three intermediate steps: One, I estimate the tariff functions from observations on payments and quantities for each tenure separately using ordinary least squares. Two, I use all information on sales and variable costs to estimate constant marginal costs as average costs. Three, using pair-wise information I estimate heterogeneous hazard rates at the percentile-tenure level and obtain percentile-to-percentile transition matrices over time.

Then, I perform the following steps iteratively starting at $\tau = 0$. One, using the empirical analog of equation 12, I estimate enforcement multipliers $\hat{\Gamma}_\tau(\cdot)$ via maximum-likelihood after parametrizing the multipliers as a logistic distribution. Two, via the empirical analogue of equation 17 and the estimated objects, I obtain estimated types $\hat{\theta}$ for each $\tau$. Three, I obtain the base marginal return functions $\hat{v}'(\cdot)$ from observations of prices and estimated types. Four, using the transition matrix, I link estimated multipliers for $s < \tau$ to quantiles of quantity at $\tau$ and repeat the process for $\tau + 1$.

Details of the estimation approach and corresponding parametric assumptions are available in Online Appendix Section OA7. Moreover, Online Appendix Section OA8 provides in Monte Carlo simulations that show that the estimation method described here can accurately recover the primitives $\{\Gamma_\tau, v(\cdot), \theta\}$ for a two-period dynamic contract.

6 Empirical Results

In this section, I first explain the definition of relationship tenure, then discuss the estimates of primitives of the model and show the data fit (both quantitatively and qualitatively). I present the results pooling all sellers together but conduct estimation at the seller-year level.

6.1 Definitions of Relationship Tenure and Estimation Sample

I make two restrictions to facilitate estimation and reduce measurement error in relationship ages. First, I require that buyers have at least one previous relationship with some seller (not necessarily those in my sample) prior to 2016.\textsuperscript{26} Second, I pool relationships ages using the following classification method and define relationship tenure between

\textsuperscript{26}I verify that this restriction is not driving the results by estimating the model with all available buyers, despite the possible measurement error in age of relationship. Overall, results are very consistent with those presented here. Results of this robustness check are available upon request.
seller \( i \) and buyer \( j \) at year \( t \) as:

\[
\text{tenure}_{ijt} = \begin{cases} 
\text{pair-age}_{ijt} & \text{if } \text{pair-age}_{ijt} < 5 \\
5 & \text{if } \text{pair-age}_{ijt} \geq 5.
\end{cases}
\]

The final sample with estimated structural model is of 24 sellers with information for 2016 and 2017 as well as 25 sellers with information for either 2016 or 2017. I consider these 73 seller-year observations on their own, but use sellers that appear on multiple years to validate fit over time.

6.2 Estimation Results

My model relies on the following seller-dependent ingredients: initial distribution of private types \( \theta \), the base return function \( v(\cdot) \), and the limited enforcement multipliers \( \Gamma, (\cdot) \) for tenure \( \tau \in \{0, 1, \ldots, 4, 5\} \).

First, Figure 2a shows the average estimated log type \( \theta \) by quantile of quantity for tenure 0. Recall that for identification, I normalized the lowest type \( \theta \) to 1. The figure shows that types increase with quantity purchased, on average across sellers, with a larger increase in the level of types for the top quantiles of quantities. Error bars show the dispersion across sellers for a given quantile.

Next, Figure 2b plots the average estimated base marginal return \( v'(\cdot) \) by quantity quantile and relationship tenure. Consistent with the model, the base marginal return function \( v'(\cdot) \) decreases as quantity increases for all. Moreover, the figure shows that older tenures experience a downward shift in their functions \( v'(\cdot) \) for a large number of quantiles, reflecting the higher levels of quantity consumption as time increases. The estimated values have an intuitive economic interpretation, as \( v'(\cdot) \) captures, for a given type, the marginal revenue for the buyer of an additional unit of the good. For the median new buyer [respectively, tenure 5], an additional unit of the good generates 2.5 [1.25] dollars of revenue for the buyer for each dollar spent on manufacturing the good by the seller (see Online Appendix Figure OA6).

Lastly, Figure 2c plots the average estimated limited enforcement multiplier \( \Gamma, (\cdot) \). It shows that, on average across-sellers, almost all new pairs are constrained, as the average multiplier \( \Gamma_0(\cdot) \) is only equal to 1 for the top 1 percent of pairs. Over time, the average multiplier approaches 1 at lower quantiles of trade, which implies the limited enforcement constraint is less restrictive.

Online Appendix Table OA9.1 shows the distribution of t-statistics for the LE multiplier at tenure 0 (\( \Gamma_0 \)) for a test against a standard model null hypothesis. Based on the significance of the parameters of estimated \( \hat{\Gamma}_0(\theta) \), I reject the null that the standard nonlinear pricing model applies in my setup for 86 percent of the markets (seller-years).

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27In Supplemental Material, I show that distribution of types per seller-year, with confidence intervals constructed via bootstrap.
6.3 Model Fit

I consider four different measures of cross-sectional model fit. First, Online Appendix Figure OA7 shows the model has good statistical fit across tenures. Second, I compare observed quantities with model predicted quantities. Quantities are constructed using the closed form solution of the seller’s first-order condition under the parametrization of \( v(\cdot) \). Online Appendix Figure OA8 plots observed quantities on the X-axis and model predicted quantities on the Y-axis. Predicted quantities match well observed ones in all tenures. Third, using predicted quantities and the incentive-compatible tariff function \( t\text{-RULE} \), I generate predicted tariffs. Online Appendix Figure OA9 plots observed tariffs on the X-axis and model generated tariffs on the Y-axis for all different tenures. Again, the model has a good performance fitting the observed tariffs. Fourth, in Figure 3a I compare the non-targeted observed cross-sectional unit price discounts by tenure to those generated by the model, which the model replicates quite well.

One may worry that the model may fail to capture within-pair dynamics, despite performing well on cross-sectional measures. For that reason, I consider a fifth validation exercise. Given that my model is estimated using cross-sectional information for each seller separately in 2016 and 2017, I can use the panel structure to verify that the primitives of the model are similar over time within pairs. Figure 3b shows the value of estimated \( \hat{\theta} \) in 2017 against the value of estimated \( \hat{\theta} \) in 2016 for pairs that are active on both years. The figure shows a good correspondence between both estimated values, with the markers overlaying the diagonal in the graph. This results therefore helps validating i)
the estimation procedure, as similar results are obtained via two independent estimation processes, and also ii) the persistency assumption for the types.

Figure 3: **Non-targeted Moments**

![Price Discounts](image1)

![Type in 2016 v. 2017](image2)

(a) Price Discounts  
(b) Type in 2016 v. 2017

**Notes:** Sub-figure a) presents a binscatter of unit prices by tenure over time, both of prices in the data and model generated prices. Model-generated unit-prices are obtained by dividing model-generated tariffs by model-generated quantities. Error bars represent 95% confidence intervals with standard errors clustered at seller-year level. Sub-figure b) shows estimated types $\theta$ in 2017 against those estimated in 2016 for buyer-seller pairs that appear on both years, which were obtained through separate seller-specific estimations for each year using cross-sectional variation alone. The dashed line represents the 45 degree line.

### 6.4 Qualitative Results

To further explore the estimated model’s implications, I consider heterogeneity in limited enforcement constraints by seller and buyer characteristics. This exercises are outside the model, but serve to provide interpretation for the enforcement parameter.

Recall that $\gamma_0(\cdot) > 0$ implies that the buyer’s limited enforcement constraint is binding. Figure 4 shows the probability that the constraint is binding at tenure 0 by differ buyer characteristics, offering qualitative differences consistent with previous literature on enforcement constraints. Consistent with the literature finding that multinational buyers are more reliable and thus less likely to see their enforcement constraint bind ex-ante (Alfaro-Urena et al., 2022), multinational buyers are less likely to have a binding enforcement constraint. Similarly, larger firms, exporters, importers, or firms in business groups are less likely to have a binding constraint. Consistent with a property-rights approach (Grossman and Hart, 1986), vertically integrated firms are less likely to have a binding constraint. Moreover, buyers that might find it hard to locate an alternative supplier, for instance, those that depend heavily on the seller as measured by their supply share, also are less likely to have a binding constraint (McMillan and Woodruff, 1999). Lastly,
distant buyers, which plausibly impose higher enforcement costs for the seller, are more likely to see their enforcement constraint bind (Antras and Foley, 2015).

Figure 5 plots the coefficients of regressions of the share of constrained buyers at tenure 0 on different sellers’ characteristics. Larger sellers measured either by total sales, total assets or size of cash holdings correlate with a lower share of constrained buyers. Similarly, sellers that export or import have a lower share of constrained buyers. This pattern is consistent with such sellers being of higher quality, and therefore, generating larger surplus for their clients at any level of quantity. Hence, buyers have lower incentive to cheat. Instead, sellers with higher levels of leverage might be too risky and offer an uncertain future for their clients. Correspondingly, higher levels of seller leverage correlate with a greater share of constrained buyers.

6.5 Performance of Alternative Models

While no theory is likely the only explanation behind an empirical phenomenon, I detail how relevant alternative models fail to match the observed dynamics in this section.

6.5.1 Seller’s Opportunistic Behavior

I have abstracted away from the possibility of seller’s opportunistic behavior. Martimort et al. (2017) developed a theory of contracts with limited enforcement, where the buyer can default on debts and the seller can act opportunistically by cheating on quality. Their model also features backloading of quantities, which increase over time, but predict increasing unit prices. The evidence regarding prices dynamics, therefore, rejects this extension of the model.

6.5.2 Customer Base

Another strand of literature (Gourio and Rudanko, 2014; Roldan-Blanco and Gilbukh, 2021; Fitzgerald et al., 2019; Piveteau, 2019) has emphasized the role of customer accumulation incentives on the dynamics of prices and quantities. These models are able to capture increases in quantities as relationships age but also predict increasing unit prices. The evidence, again, would reject this type of extension of the model.

6.5.3 Pair-specific Productivity Improvements

Productivity improvements could drive the growth in quantities and decline in prices (Heise, 2019). If bilateral trade becomes more efficient as relationships age, pair-specific marginal costs decrease, leading to lower prices. As discussed in Online Appendix Section OA3, common-law multinational do not experience price discounts over time. Given that the empirical evidence highlights the productivity effects of becoming the supplier of a

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Also, interviews with managers in the field suggested that supplier misbehavior was not a main issue of concern, but rather opportunistic behavior on the buyer’s side. Furthermore, the trade-credit literature (e.g. Smith, 1987; Breza and Liberman, 2017) argue that trade-credit itself serves a mechanism to guarantee product quality.
**Figure 4: Enforcement Constraints and Buyer Characteristics**

Notes: These figures present heterogeneity of estimated limited enforcement multipliers by buyer’s characteristics. The figures show the share of buyers in each group with positive enforcement constraint $\gamma_0(\cdot)$ in tenure 0. Classification for multinational is from the firm registry. A firm is large if they are in the top 25 of sales from the set of buyers. Firms are vertically integrated if they have any common owner with at least 1% of shares in each firm. Supply share is defined dividing total expenditure on seller by total expenditures in all intermediate inputs. Buyer is classified as exporter if they report at least $5,000 USD of exports and importer if they report at least $5,000 USD of imports. Distance between headquarters is calculated as KM. between neighborhoods as the crow flies. I classify a buyer as part of a business group if they have at least link with another firm in the economy through an shareholder that owns at least 1% shares in each firm.

**Figure 5: Correlation Seller Characteristics and Share of Constrained Buyers**

Notes: This figure plots the estimated coefficients of a regression of the share of constrained buyers for each seller-year on different seller’s characteristics. Sales refer to total sales. I classify a seller as an exporter if they report exports of at least $5,000 USD and as an importer if they report imports of at least $5,000 USD. Cash holdings, total debt, and total assets are obtained through the financial statements. Leverage is estimated as total debt over total assets.
multinational (Alfaro-Urena et al., 2022), and more so to such multinational, it seems unlikely that the price dynamics explained solely by productivity improvements would fail to find those improvements when trading with multinationals.

6.5.4 Learning about Reliability

Some recent in international trade (Macchiavello and Morjaria, 2015; Antras and Foley, 2015; Araujo et al., 2016; Monarch and Schmidt-Eisenlohr, 2017), as well as contributions in theory (Ghosh and Ray, 1996; Kranton, 1996; Ghosh and Ray, 2016), have highlighted the possible role of learning about an agent’s reliability in explaining price and quantity dynamics. Suppose the seller learns about the buyer’s reliability over repeated interactions, then the optimal price decreases because the risk of default decreases. In results available in Supplemental Material, I formally consider the model used in the previous literature and estimate it using the main data in the paper. I find the model is not able to reconcile the patterns in the data. The associated price discounts imply default rates at least 20 times larger than those observed in the data. Moreover, an estimated model is not able to recover the observed price discounts in relationships age, neither by allowing default rates to be free nor calibrating them to observed rates. Lastly, the limited enforcement model offers better statistical fit.\footnote{It is important to highlight that some of literature on learning (e.g., Ghosh and Ray, 1996; Kranton, 1996; Ghosh and Ray, 2016) require that all buyers need to be incentivized to repay their debts. Moreover, this literature relaxes the assumption that the seller has commitment on the long-term contract. Despite these additional realistic features, the models would also require higher than observed defaults to explain the dynamics. At the same time, as the asymmetric information is on the likelihood of default and not the actual size of the surplus, these models would predict no unit-price differences across buyers.}

6.5.5 Price Dynamics in Estimated Alternative Models

In Online Appendix OA10, I consider the empirical fit of alternative models in terms of the price dynamics in the cross-section. The models I consider are: standard model, learning about reliability, and the estimated limited enforcement model where I shut down the promises captured through past LE multipliers. As mentioned above, the limited enforcement model fits the data well. However, the standard model and the learning model predicts weakly increasing prices at the beginning of the relationship. Shutting down the promises in the estimated limited enforcement model diminishes model fit.

7 Welfare and Counterfactuals

In this section, I employ the estimated model to calculate the efficiency of the relationships over time. Furthermore, I consider the welfare performance of alternative pricing and enforcement schemes. In particular, I consider three margins: i) perfect enforcement with full price discrimination, ii) limited enforcement with uniform pricing, and iii) perfect enforcement with uniform pricing.
7.1 Efficiency Relative to First-Best

Under the parametrization $v(q) = kq^\beta$, first-best quantities for each pair is given by:

$$q^{fb}(\theta) = \left(\frac{k\theta}{c}\right)^{1/(1-\beta)}.$$  \hspace{1cm} (19)

Moreover, total surplus is a function of buyer’s type $\theta$, quantity $q$ and seller’s marginal cost $c$: $\text{Surplus}(\theta, q, c) = \theta kq^\beta - cq$. Hence, static efficiency of allocation $q$ for buyer type $\theta$ is defined as:

$$\text{Efficiency}(\theta, q) = \frac{\text{Surplus}(\theta, q)}{\max_q \text{Surplus}(\theta, q)}.$$

Figure 6a plots the average efficiency for each tenure across quantity deciles, averaging over all pairs, excluding tenure 1 and 3 for clearer visualization. The figure shows that new relationships are severely constrained, with the median buyer trading only at around 30% their optimal level. Yet, as relationship age, efficiency increases, with the median buyer now trading at 60% optimal levels at tenure 2, 75% at tenure 4, and north of 80% at tenure 5. Besides indicating the evolution of efficiency over time, the figure also shows a great heterogeneity in traded efficiency within relationship age: partners trading little have greater distortions than partners trading more intensively.

Of course, this characterization of efficiency might be too strict if the majority of trade is channelled through large buyers. To account for the intensity-inclusive efficiency, I study instead the weighted average efficiency of all transactions per seller. To accurately capture potential efficiency losses, I construct the weights using share of total (potential) efficient quantities at a given tenure. Under this measure, I find that total output is inefficient early on but converges towards efficiency in the medium and long term. In Table 1, I report the share of sellers that are trading at efficient levels, both in average total output and with the average buyer.\footnote{I test for seller-level efficiency via 30 bootstrap simulations and call seller’s output efficient if the 90th percentile of weighted surplus is within 1% of efficiency.} Only 5% of sellers are trading efficiently with new buyers. Efficiency increases quickly, with 70% of sellers trading efficiently by tenure 2. In the longer term, 84% of sellers transact with their buyers at efficient levels.

I next consider surplus division in Figure 6b. The figure shows the average buyer share of surplus, across sellers, by bins over quantiles of quantity purchased at different tenures. Sellers capture the majority of the surplus. The median buyer in any tenure captures around 30 percent of the surplus that is generated. In line with the nonlinear pricing scheme, buyers trading more intensively capture also larger shares of surplus, up to 50 percent. Yet, the smallest buyers may capture less than 10 percent of the pie.
Figure 6: **Efficiency and Buyer Surplus**

![Efficiency and Buyer Surplus Graph](image)

(a) Efficiency  
(b) Buyer Surplus

*Notes:* Sub-figure a) presents average efficiency by quantile of quantity and tenure over all sellers. Error bars show dispersion of ±1.96 standard errors for each quantile across sellers. Sub-figure b) shows buyer share of surplus for quantile of quantity and tenure. Error bars show ±1.96 standard errors, clustered at the seller-year level.

Table 1: % **Share of Sellers with Efficient Trade**

<table>
<thead>
<tr>
<th>Tenure</th>
<th>Weighted</th>
<th>Unweighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.48</td>
<td>5.48</td>
</tr>
<tr>
<td>1</td>
<td>41.10</td>
<td>21.92</td>
</tr>
<tr>
<td>2</td>
<td>69.86</td>
<td>32.88</td>
</tr>
<tr>
<td>3</td>
<td>79.45</td>
<td>36.99</td>
</tr>
<tr>
<td>4</td>
<td>75.00</td>
<td>37.50</td>
</tr>
<tr>
<td>5</td>
<td>84.29</td>
<td>30.00</td>
</tr>
</tbody>
</table>

*Notes:* This table reports the share of sellers that trade efficiently. The first measure (Weighted) computes the share of sellers whose weighted average output cannot be rejected to be different from the efficient output at the 10% level. The weights are constructed over potential output for each seller-tenure. The second measure (Unweighted) computes the share of sellers for which the surplus created by the average buyer cannot be rejected to be different from efficient at the 10% level.

### 7.2 Counterfactuals

Next, I use the estimated model to assess the implications of i) improving enforcement of the trade-credit contracts and ii) enforcing current legislation forbidding price discrimination on otherwise identical transactions (e.g., transactions purchasing the same quantity of the same product in a specific time period). I do so in three counterfactuals: 1) maintain price discrimination but eliminate limited enforcement, 2) maintain limited enforcement but eliminate price discrimination, and 3) eliminate both limited enforcement and price discrimination.

**Counterfactual 1: Perfect Enforcement + Price Discrimination**

A natural question that arises is what would the generated surplus and corresponding surplus shares be in a world of perfect enforcement of contracts. To obtain quantities...
under perfect enforcement, I use the distribution of types at tenures $\tau$ and equation Q-
CES, setting $\Gamma_\tau(\cdot)$ to 1 and $\gamma_\tau(\cdot)$ to 0, as well as $\Gamma_s(\cdot)$ to 1 for $s < \tau$.

**Counterfactual 2: Limited Enforcement + Uniform Pricing**

Written law in Ecuador, the European Union, and the US forbid price discrimination that applies differential treatment to customers performing an otherwise equivalent transaction, including possibly preferential treatment due to tenure.\(^{31}\) This counterfactual studies the welfare effects of a policy that enforces uniform pricing but keeps the limited enforcement regime active.

Under the assumed base return function, the optimal uniform price is $p^l = c/\beta$ for any quantity. The corresponding type $\theta$’s demand is given by $q^l(\theta) = (\alpha\beta\theta/p^l)^{1/(1-\beta)}$. This stationary menu will be insufficient for some enforcement constraints. Given exogenous hazard rates $X(\theta)$, the stationary enforcement constraint will be given by:

$$\delta(1 - X(\theta)) \geq \beta \quad \text{(L-LE)}$$

which indicates that the rate of return captured by $\beta$ has to be smaller than the buyer-specific discount rate. Notice that this limited enforcement constraint will hold for any other uniform price, so buyers who are willing to default at the optimal uniform price $p^l$ will also be willing to default at any other alternative uniform price $p^l_a$.

Under a monotonicity assumption on $X(\theta)$,\(^{32}\) the seller will set a minimum quantity $q^l$ that the buyer needs to announce in order to be served. In particular, it will only serve $q(\theta) \geq q^l$, where $q^l = \min\{q^l(\theta) | \delta(1 - X(\theta)) \geq \beta\}$. In the counterfactual exercise, I set their quantities to zero to those $\theta$ with $q^l(\theta) < q^l$.\(^{33}\)

**Counterfactual 3: Perfect Enforcement + Uniform Pricing**

Lastly, I consider optimal uniform pricing under perfect enforcement. I use quantities and prices as in counterfactual 2 above. However, as buyers are precluded from the possibility of default, the seller serves all buyers. Thus, no quantity is set to zero.

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\(^{31}\)In Ecuador, Art. 9 of Ley Orgánica de Regulación y Control del Poder de Mercado. In the EU, Art. 102(c) of Treaty on the Functioning of the European Union (ex of Art. 82(c) of EC Treaty). In the US, Section 2(a) of the Robinson-Patman Act. In practice, only the EU has enforced such a law in court. See, for instance, the cases Hoffmann-La Roche v. Commission and Manufacture française des pneumatiques Michelin v Commission. In the US, some variants of preferential pricing (such as loyalty discounts in multiproduct markets) have been upheld in court. See, for instance, cases LePage’s v 3M and SmithKline v Eh Lilly. Moreover, in the US, discounts below cost are seen as anticompetitive (see Eisai Inc. v. Sanofi-Aventis U.S., LLC). In Ecuador, no cases have been brought to court regarding the specific Art 9.

\(^{32}\)The monotonicity on the hazard rate $X'(\theta) < 0$ is observed in the data.

\(^{33}\)In this counterfactual exercise, I use an additional assumption: buyers demand the optimal level of quantity that is consistent with prices and full enforcement.
7.2.1 Discussion of Counterfactual Results

Table 2 presents the results. The table shows the average surplus in the counterfactual scenario as percentage of baseline for each percentile group in quantity and tenure.

Panel A shows the results for counterfactual 1 (nonlinear pricing with perfect enforcement). The policy exercise generates an inter-temporal trade-off. Fixing enforcement generates massive gains for middle and lower types in the early stages of the relationship. That is, under weak enforcement, the seller is forced to generate further downward distortions when buyers can default on trade. Fixing enforcement alone would increase surplus for 75% of the buyers in tenure 0 and 1. However, as relationships age, contract enforcement distortions become of second order. By tenure 3 and onward, the limited enforcement contracts actually help discipline the market power of the seller. Essentially for all buyers, fixing enforcement would decrease the generated surplus in old relationships. This is due to the fact that the seller increases quantities over time to incentivize the payments of debt from the buyer side. In the long-term, the threat of default is sufficient to overcome sellers’ market power.

Panel B presents the results for counterfactual 2 (uniform pricing with limited enforcement). Across time and types, the surplus is between 0 to 40 percent of the baseline surplus. The surprisingly low performance of this alternative regime is explained by the large share of buyers that would be excluded from trade. Some buyers cannot credibly commit to repaying their debts and the seller cannot use dynamic incentives to discipline their behavior. Thus, in the presence of limited enforcement, the seller’s ability to price discriminate actually improves the situation for both buyers and sellers, by increasing the share of buyers that can be credibly incentivized not to default.

Panel C reports the results for counterfactual 3 (uniform pricing with perfect enforcement). The table shows that surplus increases relative to baseline, except for the highest types. Welfare gains are concentrated in the lowest types (who see gains of up to 46,000%), although even median types also see large increases (from 12% up to 8,000%). Given that this counterfactual allows the seller to choose the profit maximizing uniform price, performance would improve the better the efforts to reduce seller market power, while contemporaneously addressing enforcement.

The counterintuitive results that solving only one friction at once may lead to welfare losses is a direct manifestation of the theory of second best (Lipsey and Lancaster, 1956). In the presence of multiple market frictions, eliminating one friction will not necessarily lead to higher welfare. In fact, in the presence of one market friction, an additional friction

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34Online Appendix Section OA11 presents additional results for the three counterfactual exercises related to buyer net return, profits, and prices.

35In results not presented here, I consider as well an alternative counterfactual regarding uniform pricing. I consider a pooling contract that offers only a unique price and quantity mix for all buyers. The seller picks the mix to prevent default of all buyers above a targeted threshold. Hence, the seller picks the profit margin and the share of defaults. In such counterfactual, I obtain again much lower surplus than in baseline.
might be necessary to reach second-best.

8 Conclusion

This paper studies how frictions distort and shape long-term relationships in the manufacturing supply chain. Through the lens of a novel theoretical model, the paper shows that allocating the bargaining power to the seller but allowing the buyer to take the goods and run has exciting implications for surplus division as well as price and quantity dynamics. In particular, I show that by introducing a limited enforcement constraint to prevent the buyer from defaulting on their debts, the seller must offer larger amounts of surplus than required in a perfect enforcement world. This generates incentives for the seller to distort trade inter-temporally by promising larger quantities and lower prices to the seller in the future to reap larger shares of profit now.

Using a unique intra-national database from Ecuador, I estimate the structural model of relational contracting with seller market power. My main estimated contribution is to quantify the efficiency of dynamic trade. I find that, at the beginning of their relationships, trade is highly inefficient. Yet, in the long-term, transacted quantities are close to full efficiency, despite the fact that the seller has market power. My results, therefore, highlight the great value created by the informal agreements between buyers and sellers. Finally, the paper shows the relatively fragility of these agreements. If a policy-maker were to push for unilateral reforms aimed at improving enforcement or at applying Ecuadorian antitrust policy, the long-term efficiency of relational contracts would not be sustained.

Yet, by addressing multiple frictions at once, welfare gains may be obtained.
References


Doval, Laura and Vasiliki Skreta, “Mechanism design with limited commitment,” Available at SSRN 3281132, 2020.


Appendix

A Point Identification of Gamma

In this section, I detail how \( \Gamma(\cdot) \) is point identified with observations of prices, quantities, and marginal cost for one seller under two assumptions. The first assumption is the parametrization of \( v(q) = kq^\beta \) for \( k > 0 \) and \( \beta \in (0, 1) \). The second assumption requires to select one state of the world: \( \{(\gamma(0) = 0, \Gamma(0) = 0); (\gamma(0) > 0, \Gamma(0) = 0); (\gamma(0) > 0, \Gamma(0) > 0)\} \). For my setting, I conjecture that the state of the world is \( (\gamma(0) > 0, \Gamma(0) > 0) \) and derive sufficient conditions for such state of the world.

A.1 Step 1: Show \( \beta \) is identified

We first show that \( \beta \) is identified from observations on prices, quantities and marginal cost for \( \tau = 0 \) in any possible state of the world. In this step, we omit subscripts \( \tau = 0 \).

Consider \( \rho(\alpha) = d\ln(\theta(\alpha)) = \theta'(\alpha)/\theta(\alpha) \). Substituting in, the key identification equation 12 becomes

\[
\frac{T'(q(\alpha)) - c}{T'(q(\alpha))} = \rho(\alpha)\left[\Gamma(\alpha) - \alpha\right] + \gamma(\alpha). \tag{20}
\]

Reordering and differentiating by \( \alpha \) yields

\[
\frac{d\left\{(T'(q(x)) - c)/T'(q(x))\rho(x)\right\}}{dx} \frac{d\left\{(\gamma(\alpha)/\rho(\alpha)\right\}}{dx} = \gamma(\alpha) - 1. \tag{21}
\]

Integrating from 0 to 1 gives

\[
\int^1_{0} \frac{d\left\{(T'(q(x)) - c)/T'(q(x))\rho(x)\right\}}{dx} dx - \int^1_{0} \frac{d\left\{(\gamma(x)/\rho(x)\right\}}{dx} dx = \int^1_{0} \gamma(x)dx - 1 = 0,
\tag{22}
\]

where the last equality follows from \( \int^1_{0} \gamma(x)dx = 1 \). Therefore,

\[
\frac{T'(q(1)) - c}{T'(q(1))\rho(1)} - \frac{T'(q(0)) - c}{T'(q(0))\rho(0)} = \frac{\gamma(1)}{\rho(1)} - \frac{\gamma(0)}{\rho(0)}, \tag{23}
\]

where by construction, \( \gamma(1) = 0 \). Reorder to obtain

\[
\gamma(0) = \frac{T'(q(0)) - c}{T'(q(0))} - \frac{T'(q(1)) - c}{T'(q(1))}. \tag{24}
\]

Use the derivative of the transfer rule to obtain \( \rho(\alpha) = \theta'(\alpha)/\theta(\alpha) = T''(q(\alpha))/T'(q(\alpha)) + A(q(\alpha)) \), where \( A(q(\alpha)) = -v''(q(x))/v'(q(x)) \). The assumed parametrization implies \( A(q) = (1 - \beta)/q \). Substituting \( \rho(\cdot) \) above gives

\[
\gamma(0) = \frac{T'(q(0)) - c}{T'(q(0))} - \frac{T'(q(1)) - c}{T'(q(1))} \left[ \frac{T''(q(0))}{T'(q(0))} + 1, -\frac{\beta}{q(0)} \right] \left[ \frac{T''(q(1))}{T'(q(1))} + 1, -\frac{\beta}{q(1)} \right]. \tag{25}
\]
which shows a unique mapping between $\beta$ and $\gamma(0)$, given knowledge of prices, quantities, and marginal cost.

A.1.1 **Case 1:** $\gamma(0) = 0$

If $\gamma(0) = 0$, equation 25 implies

$$1 - \beta = \left[ \frac{T'(q(1)) - c T''(q(0))}{T'(q(1)) - T''(q(0))} \right] \left[ \frac{T''(q(0))}{T'(q(0))} - \frac{T''(q(1))}{T'(q(1))} \right]^{-1}.$$

(26)

Therefore, $\beta$ is identified from observations in prices, quantities, and marginal cost when $\gamma(0) = 0$.

A.1.2 **Case 2:** $\gamma(0) > 0$ and $\Gamma(0) = 0$

For $\gamma(0) > 0$, substitute 25 in 12 evaluated at $\alpha = 0$, use $\rho(0)$ and rearrange to obtain:

$$\frac{T'(q(1)) - c}{T'(q(1))} \frac{1}{\frac{T''(q(1))}{T'(q(1))} + \frac{1-\beta}{q(1)}} = \Gamma(0)$$

(27)

If $\Gamma(\alpha)$ does not have a mass point at $\alpha = 0$, then $\Gamma(0) = 0$. If $T'(q(1)) \neq c$, equation 27 implies:

$$1 - \beta = -\frac{T''(q(0)) q(0)}{T'(q(0))}.$$

(28)

So $\beta$ is identified when $\Gamma(0) = 0$ if $T'(q(1)) \neq c$. Given observations of prices and marginal costs, this last condition can be verified in the data to hold.

A.1.3 **Case 3:** $\gamma(0) > 0$ and $\Gamma(0) = \gamma(0)$

If $\Gamma(\alpha)$ has a mass point at $\alpha = 0$, then $\Gamma(0) = \gamma(0)$. Substitute 25 into 27 and rearrange to obtain:

$$1 - \beta = \left[ \frac{T'(q(0)) - c T''(q(1))}{T'(q(0)) - T''(q(1))} - \frac{T'(q(1)) - c}{T'(q(1))} \left( 1 + \frac{T''(q(0))}{T'(q(0))} \right) \right] \left[ \frac{T'(q(1)) - c}{q(0)T'(q(1))} - \frac{T'(q(0)) - c}{q(1)T'(q(0))} \right]^{-1}.$$

(29)

Therefore, $\beta$ is identified with observations on prices, quantities, and marginal costs.

A.1.4 A Conjecture

By inspection of the solution to $\Gamma(\theta(0))$ (available in Supplemental Material Section SM2), I conjecture that Case 3, i.e., $\Gamma_0(0) > 0$, is the relevant one for my setting.

For distributions with mass points at $\theta(0) = \theta$, so $F(\theta) > 0$, a sufficient condition for $\Gamma(0)$ to be positive is:

$$\theta \beta k q(0)^{\beta-1} > c,$$

(30)

Recall that the measure $\gamma(\cdot)$ may have discrete jumps at some points. And specifically, I consider measures that may have discrete jumps at $\alpha = 0$. 

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which is equivalent to say that serving the lowest type at observed quantity \( q(0) \) is socially desirable. In practical terms, as \( T'(q(0)) = \theta(0)\beta q(0)^{\beta-1} \), then the condition for identification is

\[
T'(q(0)) > c,
\]

when the distribution of \( \theta \) is assumed to have a mass point at \( \theta^* \).

For cases with continuous distributions, so \( F(\theta) = 0, \Gamma(0) > 0 \) will always be positive, as long as \( \theta^* \) is finite and \( c \) is positive.

### A.2 Step 2: Show \( \Gamma_0 \) is identified from \( \beta \)

Consider equation 13 and use the parametrized version of \( \rho_0(\alpha) \):

\[
\Xi_0(\alpha) = \alpha + \frac{T'_0(q_0(\alpha)) - c}{T'_0(q_0(\alpha))} \left[ \frac{T''_0(q_0(\alpha))}{T'_0(q_0(\alpha))} + \frac{1 - \beta}{q_0(\alpha)} \right]^{-1}.
\]

As \( c, T'_0(\cdot), T''_0(\cdot), q_0(\cdot) \) are known, \( \Xi_0(\alpha) \) is identified up to \( \beta \). As \( \beta \) is identified from observations of prices, quantities and marginal cost, then \( \Xi_0(\alpha) \) is identified.

Then, \( \Gamma_0(\alpha) \) is identified from the solution to the differential equation

\[
\gamma_0(\alpha) + \Gamma_0(\alpha) \left[ \frac{T'_0(q_0(\alpha))}{T'_0(q_0(\alpha))} + \frac{1 - \beta}{q_0(\alpha)} \right]^{-1} = \Xi_0(\alpha) \left[ \frac{T''_0(q_0(\alpha))}{T'_0(q_0(\alpha))} + \frac{1 - \beta}{q_0(\alpha)} \right]^{-1},
\]

using the boundary condition \( \Gamma_0(1) = 1 \), and the fact that \( T''_0(\cdot), T'_0(\cdot), q_0(\cdot), \beta \) are known or identified.

### A.3 Step 3: Show \( \Gamma_\tau \) is identified from \( \beta \) and \( \Gamma_s \) for \( s < \tau \)

Start recursively from \( \tau = 1 \). With knowledge of \( \Gamma_s(\cdot) \) for \( s < \tau \) and \( \beta \), note that

\[
\Xi_\tau(\alpha) = \alpha + \sum_{s=0}^{\tau-1} (1 - \Gamma_s(\alpha)) + \frac{T'_\tau(q_\tau(\alpha)) - c}{T'_\tau(q_\tau(\alpha))} \left[ \frac{T''_\tau(q_\tau(\alpha))}{T'_\tau(q_\tau(\alpha))} + \frac{1 - \beta}{q_\tau(\alpha)} \right]^{-1}
\]

is identified as \( \Gamma_s(\cdot), c, T'_\tau(\cdot), T''_\tau(\cdot), q_\tau(\cdot), \beta \) are known or identified.

Then, \( \Gamma_\tau(\alpha) \) is identified from the solution to the differential equation

\[
\gamma_\tau(\alpha) + \Gamma_\tau(\alpha) \left[ \frac{T''_\tau(q_\tau(\alpha))}{T'_\tau(q_\tau(\alpha))} + \frac{1 - \beta}{q_\tau(\alpha)} \right]^{-1} = \Xi_\tau(\alpha) \left[ \frac{T''_\tau(q_\tau(\alpha))}{T'_\tau(q_\tau(\alpha))} + \frac{1 - \beta}{q_\tau(\alpha)} \right]^{-1},
\]

using the boundary condition \( \Gamma_\tau(1) = 1 \), and the fact that \( T''_\tau(\cdot), T'_\tau(\cdot), q_\tau(\cdot), \beta \) are known or identified.